AREA BOUNDED BY THE Y-AXIS

2 Calculate the area of the region bounded by the curve $y = \sqrt{x}$, the y-axis and the line $y = 3$.

$y = \sqrt{x} \iff x = y^2$.

Also the curve $y = \sqrt{x}$ crosses the y-axis at $\begin{cases} x = 0. \\ y = 0. \end{cases}$

So the area is $\int_0^3 y^2 \, dx$

$$\int_0^3 y^2 \, dx = \left[\frac{y^3}{3}\right]_0^3 = 9 \text{ units}^2$$

3 The area of the region bounded by the curve $y = \sqrt[3]{x}$ and the line $y = 2$ is given by:

A $\int_0^2 y \, dy$ **B** $\int_0^8 y \, dy$ **C** $\int_0^2 y^3 \, dy$ **D** $\int_0^8 y^3 \, dy$

$y = \sqrt[3]{x} \iff x = y^3$

Also the curve $y = \sqrt[3]{x}$ crosses the y-axis at $y = 0$.

So the area is $\int_0^2 y^3 \, dy$

Response $\boxed{C}$

AREA BOUNDED BY THE Y-AXIS

4 Calculate the area of the region bounded by the curve $y = \frac{1}{x^2}$, the y-axis and the lines $y = 1$ and $y = 9$.

$$y = \frac{1}{x^2} \iff x^2 = \frac{1}{y} \iff x = \frac{1}{\sqrt{y}}.$$

So the area is $\displaystyle\int_1^9 \frac{1}{\sqrt{y}}\,dy = \int_1^9 y^{-1/2}\,dy = \left[\frac{y^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right]_1^9$

$$= \left[\frac{y^{0.5}}{0.5}\right]_1^9 = \left[2\sqrt{y}\right]_1^9$$

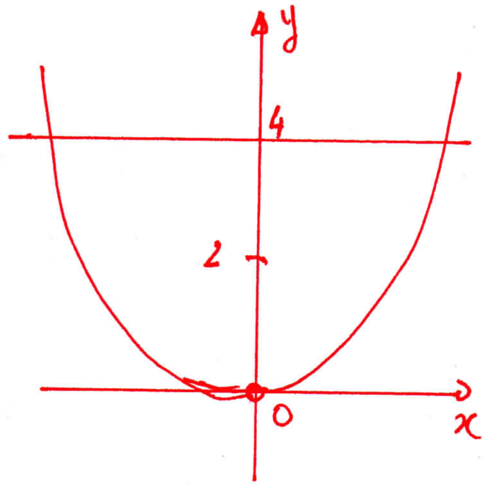
$$= 2\sqrt{9} - 2\sqrt{1} = 2 \times 3 - 2$$

$$= 4 \text{ units}^2$$

5 Calculate the area of the region bounded by the curve $y = x^2$ and the line $y = 4$.

$y = x^2 \iff x = \sqrt{y}$.

So the area is $\displaystyle 2\int_0^4 \sqrt{y}\,dy$.



$$2\int_0^4 \sqrt{y}\,dy = 2\left[\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]_0^4 = 2\left[\frac{x^{3/2}}{3/2}\right]_0^4$$

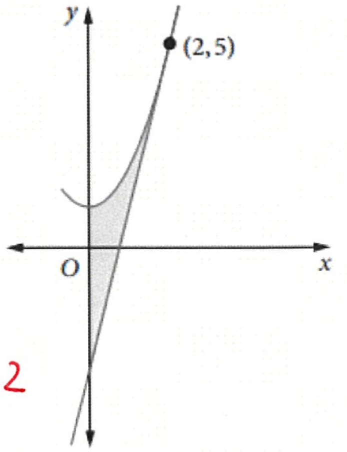
$$= \frac{4}{3}\left[x^{3/2}\right]_0^4 = \frac{4}{3} \times 4^{3/2}$$

$$= \frac{4}{3} \times 2^3 = \frac{4}{3} \times 8 = \frac{32}{3} = 10\frac{2}{3} \text{ units}^2$$

AREA BOUNDED BY THE Y-AXIS

6 **(a)** Show that the equation of the tangent to the parabola $y = x^2 + 1$ at the point where $x = 2$ is $y = 4x - 3$.

(b) Hence find the area enclosed by the parabola, the tangent and the y-axis.



a) The derivative of $y = x^2+1$ is $\frac{dy}{dx} = 2x$

At $x=2$, $\frac{dy}{dx} = 2 \times 2 = 4$

which is the gradient of the tangent at $x=2$

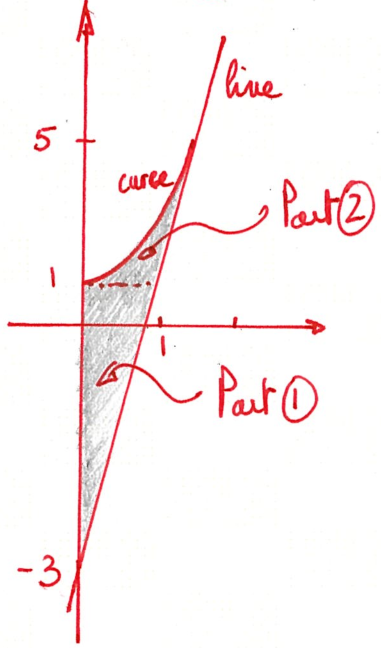
This tangent passes through $(2,5)$

Hence its equation is given by the gradient-point formula,

$y - y_0 = m(x - x_0) \Rightarrow y - 5 = 4(x-2)$

$\Leftrightarrow y = 4x - 8 + 5$ $\boxed{\text{i.e. } y = 4x-3}$

b) we divide the area into 2 parts, Part 1 and Part 2



Area Part 1 $= \frac{1 \times 4}{2} = 2$ units^2

Area Part 2 $= \int_1^5 (\text{line} - \text{curve})\, dy$.

$y = 4x - 3$ so $4x = y+3 \Rightarrow x = \frac{y+3}{4}$

$y = x^2+1$ so $x^2 = y-1$ so $x = \sqrt{y-1}$ for $1 < y < 5$

$\therefore$ Area Part 2 $= \int_1^5 \left(\frac{y+3}{4}\right) - \sqrt{y-1}\, dy$

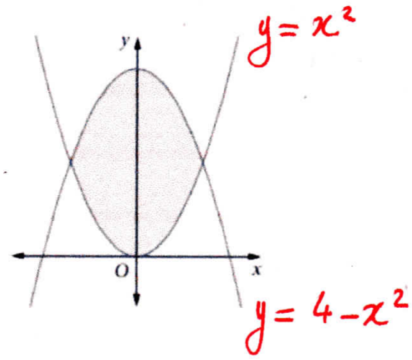
$= \frac{1}{4}\int_1^5 y+3\, dy - \int_1^5 \sqrt{y-1}\, dy = \frac{1}{4}\left[\frac{y^2}{2} + 3y\right]_1^5 - \left[\frac{(y-1)^{3/2}}{3/2}\right]_1^5$

$= \frac{1}{4}\left(\frac{25}{2} + 15 - \frac{1}{2} - 3\right) - \left(\frac{2 \times 8}{3} - \frac{2}{3} \times 0\right) = 6 - \frac{16}{3} = \frac{2}{3}$

Total 1 + 2 $= 2 + \frac{2}{3} = \frac{8}{3}$ units^2

AREA BOUNDED BY THE Y-AXIS

8 **(a)** Calculate the area of the region bounded by the parabolas $y = x^2$ and $y = 4 - x^2$.
(b) Calculate the area of the region bounded by the x-axis and the parabolas $y = x^2$ and $y = 4 - x^2$.



a) The parabolas intersect where $x^2 = 4 - x^2$

i.e. $2x^2 = 4 \iff x^2 = 2$

$\iff x = \pm\sqrt{2}$

So the area is $2 \times \int_0^{\sqrt{2}} \left[(4 - x^2) - x^2\right] dx$

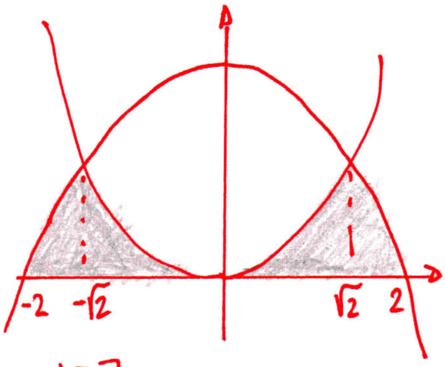
$$= 2 \times \int_0^{\sqrt{2}} 4 - 2x^2 \, dx$$

$$= 2 \left[4x - 2\frac{x^3}{3}\right]_0^{\sqrt{2}}$$

$$= 2\left(4 \times \sqrt{2} - \frac{4\sqrt{2}}{3}\right) = 2 \times 4\sqrt{2}\left(\frac{2}{3}\right) = \frac{16}{3}\sqrt{2} \text{ units}^2$$

b) The parabola $y = 4 - x^2$ intersect the x-axis at $x = \pm 2$.

So the area is $2\left[\int_0^{\sqrt{2}} x^2 \, dx + \int_{\sqrt{2}}^{2} (4 - x^2) \, dx\right]$



$$= 2\left[\left[\frac{x^3}{3}\right]_0^{\sqrt{2}} + \left[4x - \frac{x^3}{3}\right]_{\sqrt{2}}^{2}\right]$$

$$= 2\left[\frac{2\sqrt{2}}{3} + \left[4 \times 2 - \frac{8}{3} - \left(4\sqrt{2} - \frac{2\sqrt{2}}{3}\right)\right]\right]$$

$$= 2\left[\frac{2\sqrt{2}}{3} + \frac{16}{3} - 2\sqrt{2}\left(2 - \frac{1}{3}\right)\right]$$

$$= 2\left[\frac{2\sqrt{2}}{3} - \frac{10\sqrt{2}}{3} + \frac{16}{3}\right] = \frac{2}{3}\left[16 - 8\sqrt{2}\right] = \frac{16(2 - \sqrt{2})}{3} \text{ units}^2$$