VECTORS IN THREE DIMENSIONS

2 For each of the points P whose coordinates are given, find:

(i) an $\underset{\sim}{i}, \underset{\sim}{j}, \underset{\sim}{k}$ representation for the position vector $\overrightarrow{OP}$ **(ii)** the magnitude of $\overrightarrow{OP}$

(iii) a unit vector in the direction of $\overrightarrow{OP}$.

(b) $P(6, 8)$ **(c)** $P(2, 2, 1)$ **(d)** $P(-3, 4, 5)$

b) i) $\overrightarrow{OP} = 6\vec{i} + 8\vec{j}$ ii) $|\overrightarrow{OP}| = \sqrt{6^2+8^2} = \sqrt{36+64} = \sqrt{100} = 10$

iii) $\dfrac{\overrightarrow{OP}}{|\overrightarrow{OP}|} = \dfrac{6\vec{i}+8\vec{j}}{10} = \dfrac{3}{5}\vec{i} + \dfrac{4}{5}\vec{j}$

c) i) $\overrightarrow{OP} = 2\vec{i} + 2\vec{j} + \vec{k}$

ii) $|\overrightarrow{OP}| = \sqrt{2^2+2^2+1^2} = \sqrt{4+4+1} = \sqrt{9} = 3$

$\dfrac{\overrightarrow{OP}}{|\overrightarrow{OP}|} = \dfrac{2\vec{i}+2\vec{j}+\vec{k}}{3} = \dfrac{2}{3}\vec{i} + \dfrac{2}{3}\vec{j} + \dfrac{1}{3}\vec{k}$

d) i) $\overrightarrow{OP} = -3\vec{i} + 4\vec{j} + 5\vec{k}$

ii) $|\overrightarrow{OP}| = \sqrt{(-3)^2+4^2+5^2} = \sqrt{9+16+25} = \sqrt{50} = 5\sqrt{2}$

iii) $\dfrac{\overrightarrow{OP}}{|\overrightarrow{OP}|} = \dfrac{-3\vec{i}+4\vec{j}+5\vec{k}}{5\sqrt{2}} = \dfrac{-3}{5\sqrt{2}}\vec{i} + \dfrac{4}{5\sqrt{2}}\vec{j} + \dfrac{1}{\sqrt{2}}\vec{k}$

or in a rationalised manner:

$\dfrac{\overrightarrow{OP}}{|\overrightarrow{OP}|} = -\dfrac{3\sqrt{2}}{10}\vec{i} + \dfrac{2\sqrt{2}}{5}\vec{j} + \dfrac{\sqrt{2}}{2}\vec{k}$

3 Given vectors $\underset{\sim}{a} = 6\underset{\sim}{i} + 3\underset{\sim}{j} - 2\underset{\sim}{k}$, $\underset{\sim}{b} = -4\underset{\sim}{i} + 3\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{c} = 2\underset{\sim}{i} + 3\underset{\sim}{k}$, write:

(b) $\underset{\sim}{a} + \underset{\sim}{b} - \underset{\sim}{c}$ **(c)** $2\underset{\sim}{c} + 3\underset{\sim}{a} - 5\underset{\sim}{b}$ **(d)** $3(\underset{\sim}{c} - \underset{\sim}{a})$

b) $\vec{a} + \vec{b} - \vec{c} = (6\vec{i} + 3\vec{j} - 2\vec{k}) + (-4\vec{i} + 3\vec{j} + \vec{k}) - (2\vec{i} + 3\vec{k})$

$= (6-4-2)\vec{i} + (3+3)\vec{j} + (-2+1-3)\vec{k}$

$= 6\vec{j} - 4\vec{k}$

c) $2\vec{c} + 3\vec{a} - 5\vec{b} = 2(2\vec{i} + 3\vec{k}) + 3(6\vec{i} + 3\vec{j} - 2\vec{k}) - 5(-4\vec{i} + 3\vec{j} + \vec{k})$

$= 4\vec{i} + 6\vec{k} + 18\vec{i} + 9\vec{j} - 6\vec{k} + 20\vec{i} - 15\vec{j} - 5\vec{k}$

$= (4+18+20)\vec{i} + (9-15)\vec{j} + (6-6-5)\vec{k}$

$= 42\vec{i} - 6\vec{j} - 5\vec{k}$

d) $3(\vec{c} - \vec{a}) = 3[(2\vec{i} + 3\vec{k}) - (6\vec{i} + 3\vec{j} - 2\vec{k})]$

$= 3[-4\vec{i} - 3\vec{j} + 5\vec{k}]$

$= -12\vec{i} - 9\vec{j} + 15\vec{k}$

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6 The position vectors of the points P, Q, R and S are respectively $4\underset{\sim}{i} + 3\underset{\sim}{j} - \underset{\sim}{k}$, $5\underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k}$, $2\underset{\sim}{i} - 2\underset{\sim}{j} - 3\underset{\sim}{k}$, $4\underset{\sim}{i} - 4\underset{\sim}{j} + 3\underset{\sim}{k}$. Show that $\overrightarrow{PQ}$ is parallel to $\overrightarrow{RS}$.

$$\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ} = -\overrightarrow{OP} + \overrightarrow{OQ}$$

$$= -(4\vec{i} + 3\vec{j} - \vec{k}) + (5\vec{i} + 2\vec{j} + 2\vec{k})$$

$$= \vec{i} - \vec{j} + 3\vec{k}$$

$$\overrightarrow{RS} = \overrightarrow{RO} + \overrightarrow{OS} = -\overrightarrow{OR} + \overrightarrow{OS}$$

$$= -(2\vec{i} - 2\vec{j} - 3\vec{k}) + (4\vec{i} - 4\vec{j} + 3\vec{k})$$

$$= 2\vec{i} - 2\vec{j} + 6\vec{k}$$

$$= 2\left[\vec{i} - \vec{j} + 3\vec{k}\right]$$

So $\overrightarrow{RS} = 2\,\overrightarrow{PQ}$

$\therefore$ $\overrightarrow{RS}$ and $\overrightarrow{PQ}$ are parallel.

9 $P(6, 3, -4)$, $Q(3, 1, 1)$ and $R(2, -1, 3)$ are the vertices of a triangle. Show that $|\overrightarrow{RP}| = 3|\overrightarrow{RQ}|$.

$\overrightarrow{RP} = \overrightarrow{RO} + \overrightarrow{OP} = -\overrightarrow{OR} + \overrightarrow{OP}$

$\quad = -(2\vec{i} - \vec{j} + 3\vec{k}) + (6\vec{i} + 3\vec{j} - 4\vec{k})$

$\quad = 4\vec{i} + 4\vec{j} - 7\vec{k}$

$\overrightarrow{RQ} = \overrightarrow{RO} + \overrightarrow{OQ} = -\overrightarrow{OR} + \overrightarrow{OQ}$

$\quad = -(2\vec{i} - \vec{j} + 3\vec{k}) + (3\vec{i} + \vec{j} + \vec{k})$

$\quad = \vec{i} + 2\vec{j} - 2\vec{k}.$

$|\overrightarrow{RP}| = \sqrt{4^2 + 4^2 + (-7)^2} = \sqrt{16 + 16 + 49} = \sqrt{81} = 9$

$|\overrightarrow{RQ}| = \sqrt{1^2 + 2^2 + (-2)^2} = \sqrt{1 + 4 + 4} = \sqrt{9} = 3$

$\therefore \quad |\overrightarrow{RP}| = 3\,|\overrightarrow{RQ}|$

10 If $\underset{\sim}{a} = 2\underset{\sim}{i} - 3\underset{\sim}{j} + \underset{\sim}{k}$, $\underset{\sim}{b} = 2\underset{\sim}{i} - 4\underset{\sim}{j} + 5\underset{\sim}{k}$, $\underset{\sim}{c} = -\underset{\sim}{i} - 4\underset{\sim}{j} + 2\underset{\sim}{k}$, find the values of p and q such that $\underset{\sim}{a} + p\underset{\sim}{b} + q\underset{\sim}{c}$ is parallel to the x-axis.

For $(\vec{a} + p\vec{b} + q\vec{c})$ to be parallel to the x-axis means that $\exists\, k \in \mathbb{R}$ such that $\vec{a} + p\vec{b} + q\vec{c} = k\vec{i}$

$$\vec{a} + p\vec{b} + q\vec{c} = (2\vec{i} - 3\vec{j} + \vec{k}) + p(2\vec{i} - 4\vec{j} + 5\vec{k}) + q(-\vec{i} - 4\vec{j} + 2\vec{k})$$

$$= (2 + 2p - q)\vec{i} + (-3 - 4p - 4q)\vec{j} + (1 + 5p + 2q)\vec{k}$$

$\therefore$ we must have $\begin{cases} -3 - 4p - 4q = 0 \\ 1 + 5p + 2q = 0 \end{cases} \Leftrightarrow \begin{cases} 4p + 4q = -3 \\ 5p + 2q = -1 \end{cases}$

$\Leftrightarrow \begin{cases} 2p + 2q = -3/2 \\ 5p + 2q = -1 \end{cases}$ By elimination, we get:

$$2p - 5p = -\frac{3}{2} - (-1) = -\frac{1}{2}$$

So $-3p = -\frac{1}{2}$ $\quad \boxed{p = \frac{1}{6}}$

and $\therefore$ $q = \frac{1}{2}(-1 - 5p) = \frac{1}{2}\left(-1 - 5 \times \frac{1}{6}\right) = -\frac{11}{12}$

$$\boxed{q = -\frac{11}{12}}$$

In that case $2 + 2p - q = 2 + 2 \times \left(\frac{1}{6}\right) - \left(-\frac{11}{12}\right) = 2 + \frac{1}{3} + \frac{11}{12} \neq 0$

which is different of 0, $\therefore$ $\vec{a} + p\vec{b} + q\vec{c} \neq \vec{0}$

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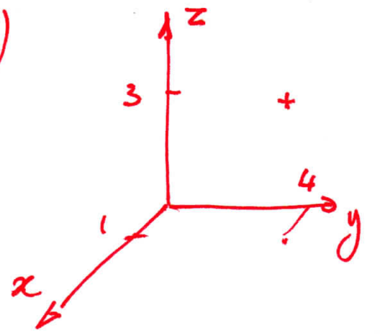
11 Find the distance of the point $P(1, 4, 3)$ from:

(a) the y–z plane **(b)** the x–z plane **(e)** the y-axis

a)



Distance from the $y-z$ plane is 1

b) Distance from the $x-z$ plane is 4

c) Using Pythagoras, this distance is:

$\sqrt{1^2 + 3^2} = \sqrt{1+9} = \sqrt{10}$.