VECTOR EQUATION OF A LINE

1 **(a)** Find the vector equation of the line through (1, 2) parallel to the vector $\underset{\sim}{i} + \underset{\sim}{j}$.

(b) Find the points corresponding to: **(i)** $\lambda = 0$ **(ii)** $\lambda = -1$ **(iii)** $\lambda = 2$.

a) $\vec{r} = \vec{r_0} + \lambda \vec{v}$

$\vec{r} = \vec{i} + 2\vec{j} + \lambda(\vec{i} + \vec{j})$

$\vec{r} = (1+\lambda)\vec{i} + (2+\lambda)\vec{j}$

b) i) $\lambda = 0$ so $\vec{r_0} = (1+0)\vec{i} + (2+0)\vec{j}$

$\vec{r} = \vec{i} + 2\vec{j}$

This point is therefore $A(1,2)$

ii) $\lambda = -1$ so $\vec{r} = (1-1)\vec{i} + (2-1)\vec{j}$

$\vec{r} = \vec{j}$

This point is therefore $(0,1)$

iii) $\lambda = 2$ so $\vec{r} = (1+2)\vec{i} + (2+2)\vec{j}$

$\vec{r} = 3\vec{i} + 4\vec{j}$

This point is therefore $(3,4)$

VECTOR EQUATION OF A LINE

2 **(a)** Find the vector equation of the line through $(-1, 4, 6)$ parallel to the vector $\underline{i} - \underline{j} + \underline{k}$.

(b) Find the points corresponding to: **(i)** $\lambda = \frac{1}{2}$ **(ii)** $\lambda = 6$ **(iii)** $\lambda = -3$.

a) $\vec{r} = (-\vec{i} + 4\vec{j} + 6\vec{k}) + \lambda(\vec{i} - \vec{j} + \vec{k})$

$\vec{r} = (-1+\lambda)\vec{i} + (4-\lambda)\vec{j} + (6+\lambda)\vec{k}$

b) i) $\lambda = 1/2$ $\quad \vec{r_{1/2}} = \left(-1+\frac{1}{2}\right)\vec{i} + \left(4-\frac{1}{2}\right)\vec{j} + \left(6+\frac{1}{2}\right)\vec{k} = -\frac{1}{2}\vec{i} + \frac{9}{2}\vec{j} + \frac{13}{2}\vec{k}$

So $A\left(-\frac{1}{2}, \frac{9}{2}, \frac{13}{2}\right)$

ii) $\lambda = 6$ $\quad \vec{r_6} = (-1+6)\vec{i} + (4-6)\vec{j} + (6+6)\vec{k} = 5\vec{i} - 2\vec{j} + 12\vec{k}$

So $B(5, -2, 12)$

iii) $\lambda = -3$ $\quad \vec{r_{-3}} = (-1-3)\vec{i} + [4-(-3)]\vec{j} + (6-3)\vec{k} = -4\vec{i} + 7\vec{j} + 3\vec{k}$

So $C(-4, 7, 3)$

3 **(a)** Find the vector equation of the line through $(4, 2)$ parallel to the line joining the points $(-1, 3)$ and $(3, 7)$.

(b) Find the points corresponding to: **(i)** $\lambda = 2$ **(ii)** $\lambda = 4$ **(iii)** $\lambda = 8$.

a) The vector joining $(-1,3)$ and $(3,7)$ is $(4,4)$ or $4(\vec{i}+\vec{j})$

$\vec{r} = 4\vec{i} + 2\vec{j} + \lambda \times 4(\vec{i}+\vec{j}) = (4+4\lambda)\cdot\vec{i} + (2+4\lambda)\vec{j}$

b) i) $\lambda = 2$ $\quad \vec{r_2} = (4+4\times2)\vec{i} + (2+4\times2)\vec{j} = 12\vec{i} + 10\vec{j}$

So the point is $(12, 10)$

ii) $\lambda = 4$ $\quad \vec{r_4} = (4+4\times4)\vec{i} + (2+4\times4)\vec{j} = 20\vec{i} + 18\vec{j}$

So the point is $(20, 18)$

iii) $\lambda = 8$ $\quad \vec{r_8} = (4+4\times8)\vec{i} + (2+4\times8)\vec{j} = 36\vec{i} + 34\vec{j}$

So the point corresponding to $\lambda = 8$ is $(36, 34)$

VECTOR EQUATION OF A LINE

4 (a) Find the vector equation of the line through (2, 3, 4) parallel to the line joining the points (0, 2, 4) and (−5, −3, 6).

(b) Find the points corresponding to: (i) $\lambda = -1$ (ii) $\lambda = 0$ (iii) $\lambda = 1$.

a) The line joining the points (0,2,4) and (−5,−3,6) is parallel to the vector (−5, −5, 2) or $-5\vec{i} - 5\vec{j} + 2\vec{k}$.

So the vector equation of that line is

$$\vec{r} = (2\vec{i} + 3\vec{j} + 4\vec{k}) + \lambda \times (-5\vec{i} - 5\vec{j} + 2\vec{k})$$

$$\vec{r} = (2 - 5\lambda)\vec{i} + (3 - 5\lambda)\vec{j} + (4 + 2\lambda)\vec{k}$$

b) i) $\lambda = -1$

$$\vec{r}_{-1} = [2 - 5\times(-1)]\vec{i} + [3 - 5\times(-1)]\vec{j} + [4 + 2\times(-1)]\vec{k}$$

$$\vec{r}_{-1} = 7\vec{i} + 8\vec{j} + 2\vec{k}$$ So A(7,8,2)

ii) $\lambda = 0$ $$\vec{r}_0 = (2 - 5\times 0)\vec{i} + (3 - 5\times 0)\vec{j} + (4 + 2\times 0)\vec{k}$$

$$\vec{r}_0 = 2\vec{i} + 3\vec{j} + 4\vec{k}$$

So the point corresponding to $\lambda = 0$ is (2, 3, 4)

iii) $\lambda = 1$ $$\vec{r}_1 = (2 - 5\times 1)\vec{i} + (3 - 5\times 1)\vec{j} + (4 + 2\times 1)\vec{k}$$

$$\vec{r}_1 = -3\vec{i} - 2\vec{j} + 6\vec{k}$$

So the point corresponding to $\lambda = 1$ is (−3, −2, 6)

VECTOR EQUATION OF A LINE

6 Find the vector equation of the line through $A(3, 5, 7)$ and $B(6, 4, 5)$.

$\overrightarrow{AB} = 3\vec{i} - \vec{j} - 2\vec{k}$

$\vec{r} = (3\vec{i} + 5\vec{j} + 7\vec{k}) + \lambda(3\vec{i} - \vec{j} - 2\vec{k})$

$\vec{r} = (3 + 3\lambda)\vec{i} + (5 - \lambda)\vec{j} + (7 - 2\lambda)\vec{k}$