FURTHER APPLICATIONS OF TRIGONOMETRIC FUNCTIONS

1 Find the derivative of $\log_e(\cos x)$.

2 Find the equation of the tangent to the curve $y = \tan x$ at $x = \frac{\pi}{4}$.

3 For $f(x) = \sin x + \cos x$ over the domain $0 \leq x \leq 2\pi$, find:

(a) $f'(x)$ **(b)** $f''(x)$ **(c)** the coordinates of any turning points
(d) the coordinates of any points of inflection **(e)** the maximum value of $f(x)$.

4 For $y = e^{\sin x}$, find the equation of the normal to the curve at the point where $x = 0$.

5 Find all the points on the graph of $y = 2\sin x + \sin^2 x$, $0 \le x \le 4\pi$, at which the tangent is horizontal.

6 The tide at a point on the WA coast can be modelled using the equation $y = a\cos nt$. At Cable Beach in WA, over two consecutive days, the average difference between high and low tides is 9.0 metres and the average time between high tide and low tide is 6.1 hours.

(a) What is the amplitude of the tide function at Cable Beach?
(b) How much time passes between successive high tides (i.e. the period) and what is the value of n?
(c) Use this information to obtain the tide function and draw its graph.
(d) If the depth of water at low tide is 0.5 metres, what is the depth of the water 1 hour after low tide?

8 Find the equation of the normal to the curve $y = \cot x$ at the point $P\left(\frac{\pi}{4}, 1\right)$.

9 If $y = e^{\operatorname{cosec} x}$, find the equation of the tangent to the curve at $x = \frac{\pi}{2}$.

10 If $y = 3\cos 4x$, prove that $\frac{d^2y}{dx^2} + 16y = 0$.