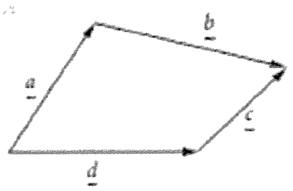
VECTORS IN TWO DIMENSIONS - CHAPTER REVIEW

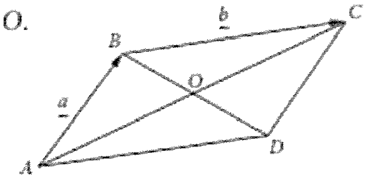
1 Four vectors, $\underset{\sim}{a}$, $\underset{\sim}{b}$, $\underset{\sim}{c}$ and $\underset{\sim}{d}$, are shown in the diagram. Which one of the following statements is true?



A $\underset{\sim}{a}+\underset{\sim}{c}=\underset{\sim}{b}+\underset{\sim}{d}$ B $\underset{\sim}{a}+\underset{\sim}{b}=\underset{\sim}{c}+\underset{\sim}{d}$

C $\underset{\sim}{a}+\underset{\sim}{b}+\underset{\sim}{c}+\underset{\sim}{d}=\underset{\sim}{0}$ D $\underset{\sim}{b}+\underset{\sim}{c}=\underset{\sim}{a}+\underset{\sim}{d}$

2 In the parallelogram $ABCD$ shown, the point of intersection of the diagonals is O. The vector $\overrightarrow{OD}$ is equal to:



A $\frac{1}{2}(\underset{\sim}{a}-\underset{\sim}{b})$ B $\frac{1}{2}(\underset{\sim}{a}+\underset{\sim}{b})$

C $\frac{1}{2}\underset{\sim}{b}-\underset{\sim}{a}$ D $\frac{1}{2}(\underset{\sim}{b}-\underset{\sim}{a})$

3 If vector $\underset{\sim}{a}$ is represented by the ordered pair $(-2, 3)$, then the vector $-3\underset{\sim}{a}$ is represented by the ordered pair:

A $(-6, 9)$ B $(-6, -9)$ C $(6, -9)$ D $(6, 9)$

4 The vector that runs from the point $(-3, 1)$ to the point $(3, -2)$ can be represented by the column vector:

A $\begin{pmatrix} 6 \\ 3 \end{pmatrix}$ B $\begin{pmatrix} 6 \\ -3 \end{pmatrix}$ C $\begin{pmatrix} -6 \\ 3 \end{pmatrix}$ D $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$

5 Which one of the following vectors is parallel to the vector $\underset{\sim}{f}=-6\underset{\sim}{i}+4\underset{\sim}{j}$?

A $\underset{\sim}{a}=24\underset{\sim}{i}-16\underset{\sim}{j}$ B $\underset{\sim}{b}=3\underset{\sim}{i}+2\underset{\sim}{j}$ C $\underset{\sim}{c}=-24\underset{\sim}{i}-16\underset{\sim}{j}$ D $\underset{\sim}{d}=-3\underset{\sim}{i}-2\underset{\sim}{j}$

6 Which one of the following vectors is parallel to the vector $\underset{\sim}{a}=-3\underset{\sim}{i}+7\underset{\sim}{j}$ and has a magnitude of $2\sqrt{58}$?

A $-24\underset{\sim}{i}+28\underset{\sim}{j}$ B $-\frac{3}{2}\underset{\sim}{i}+\frac{7}{2}\underset{\sim}{j}$ C $3\underset{\sim}{i}-7\underset{\sim}{j}$ D $-6\underset{\sim}{i}+14\underset{\sim}{j}$

VECTORS IN TWO DIMENSIONS - CHAPTER REVIEW

7 Given position vectors $\overrightarrow{OA} = -3\underset{\sim}{i} + 4\underset{\sim}{j}$ and $\overrightarrow{OB} = 4\underset{\sim}{i} + 3\underset{\sim}{j}$, what is the value of $\left|\overrightarrow{AB}\right|$?

A $\sqrt{2}$ B $5\sqrt{2}$ C $2\sqrt{5}$ D $7\sqrt{2}$

8 If $\underset{\sim}{a} = -4\underset{\sim}{i} + 2\underset{\sim}{j}$ and $\underset{\sim}{b} = \underset{\sim}{i} - 4\underset{\sim}{j}$, then $2\underset{\sim}{a} - \underset{\sim}{b}$ is:

A $-5\underset{\sim}{i} - 2\underset{\sim}{j}$ B $-6\underset{\sim}{i} + 10\underset{\sim}{j}$ C $-9\underset{\sim}{i} + 8\underset{\sim}{j}$ D $-10\underset{\sim}{i} - 4\underset{\sim}{j}$

9 What is the magnitude of the vector $\underset{\sim}{a} = 4\underset{\sim}{i} - 2\underset{\sim}{j}$?

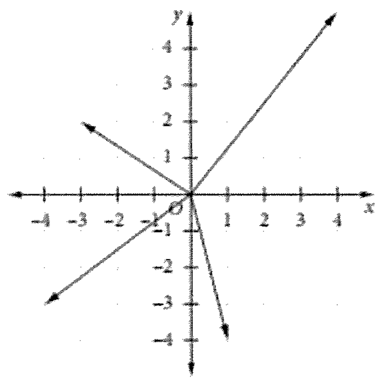
A 2 B $2\sqrt{3}$ C $2\sqrt{5}$ D 20

10 Which of the following vectors is parallel to the vector $2\underset{\sim}{i} + 3\underset{\sim}{j}$ and has a magnitude of $2\sqrt{13}$?

A $-4\underset{\sim}{i} + 6\underset{\sim}{j}$ B $4\underset{\sim}{i} - 6\underset{\sim}{j}$ C $6\underset{\sim}{i} + 9\underset{\sim}{j}$ D $-4\underset{\sim}{i} - 6\underset{\sim}{j}$

19 Label the following vectors that have been drawn on the Cartesian plane:

- $\underset{\sim}{a}$ the position vector of $(-3, 2)$
- $\overrightarrow{OB}$ where B is $\begin{pmatrix} -4 \\ -3 \end{pmatrix}$
- $\underset{\sim}{c}$ the position vector of $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$
- $\overrightarrow{OD}$ where D is $(1, -4)$



VECTORS IN TWO DIMENSIONS - CHAPTER REVIEW

26 Find the exact values of the unknown pronumerals in the following vector equations.

(a) $(2a-3b)\underset{\sim}{i}-2b\underset{\sim}{j}=5\underset{\sim}{i}-12\underset{\sim}{j}$

(b) $(2f+5)\underset{\sim}{i}+(8-7g)\underset{\sim}{j}=f\left(3\underset{\sim}{i}-2\underset{\sim}{j}\right)+2g\left(\underset{\sim}{i}+4\underset{\sim}{j}\right)$

(c) $\left(a^2-9a\right)\underset{\sim}{i}+\left(2b^3+1\right)\underset{\sim}{j}=10\underset{\sim}{i}-5\underset{\sim}{j}$ (list multiple solutions)

27 Consider the vector $\underset{\sim}{a}=-9\underset{\sim}{i}-3\underset{\sim}{j}$.

(a) Find $\hat{\underset{\sim}{a}}$.

(b) Find the vector $\underset{\sim}{b}$ in the direction of $\underset{\sim}{a}$ with a magnitude of 5.

28 Find the scalar product $\underset{\sim}{a} \bullet \underset{\sim}{b}$, given the following pairs of vectors.

(a) $\underset{\sim}{a} = -4\underset{\sim}{i} + \underset{\sim}{j}$ and $\underset{\sim}{b} = 2\underset{\sim}{i} + 7\underset{\sim}{j}$

(b) $\underset{\sim}{a} = 3\underset{\sim}{i} - 7\underset{\sim}{j}$ and $\underset{\sim}{b} = 6\underset{\sim}{i} - \underset{\sim}{j}$

29 Calculate the scalar product and hence show that the vectors $\underset{\sim}{a} = -3\underset{\sim}{i} + 5\underset{\sim}{j}$ and $\underset{\sim}{b} = 10\underset{\sim}{i} + 6\underset{\sim}{j}$ are perpendicular.

30 For each of the following pairs of vectors, find the scalar projection of $\underset{\sim}{a}$ onto $\underset{\sim}{b}$.

(a) $\underset{\sim}{a} = 3\underset{\sim}{i} - 4\underset{\sim}{j}$ and $\underset{\sim}{b} = 6\underset{\sim}{i} + 3\underset{\sim}{j}$

(b) $\underset{\sim}{a} = -5\underset{\sim}{i} + 2\underset{\sim}{j}$ and $\underset{\sim}{b} = \underset{\sim}{i} - 7\underset{\sim}{j}$

VECTORS IN TWO DIMENSIONS - CHAPTER REVIEW

31 For $\underset{\sim}{a} = 2\underset{\sim}{i} - 5\underset{\sim}{j}$ and $\underset{\sim}{b} = 4\underset{\sim}{i} + \underset{\sim}{j}$, find:

(a) the vector projection of $\underset{\sim}{a}$ onto $\underset{\sim}{b}$

(b) the vector projection of $\underset{\sim}{a}$ perpendicular to $\underset{\sim}{b}$.

32 The points A, B and C have coordinates $(2, -5)$, $(5, 9)$ and $(-9, 12)$ respectively.

(a) Find the vectors $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{AC}$ in column vector form.

(b) Find $\left|\overrightarrow{AB}\right|$, $\left|\overrightarrow{BC}\right|$ and $\left|\overrightarrow{AC}\right|$.

(c) Show that ΔABC is an isosceles triangle.

(d) Find the coordinates of a point D such that $ABCD$ forms a rhombus.

(e) Find the coordinates of the point of intersection of the diagonals of the rhombus $ABCD$.

33 (a) If $\underset{\sim}{a} = -4e\underset{\sim}{i} + 2e\underset{\sim}{j}$, $e > 0$ and $|\underset{\sim}{a}|^2 = 40$, find the exact value of e. (b) Hence, find $\hat{\underset{\sim}{a}}$.

(c) Find the vector $\underset{\sim}{b}$ that is parallel to $\hat{\underset{\sim}{a}}$ with $|\underset{\sim}{b}| = 10$.

(d) If $\underset{\sim}{c} = 4f\underset{\sim}{i} - 3f\underset{\sim}{j}$, $f > 0$ and $|\underset{\sim}{c}|^2 = 250$, find the exact value of f. (e) Hence find $\hat{\underset{\sim}{c}}$.

(f) Find the vector $\underset{\sim}{d}$ in the direction of $\hat{\underset{\sim}{c}}$ where $|\underset{\sim}{d}|^2 = 20$. (g) Find $\underset{\sim}{b} - \underset{\sim}{d}$.

VECTORS IN TWO DIMENSIONS - CHAPTER REVIEW

34 Consider two vectors $\underset{\sim}{a} = 2\underset{\sim}{i} - 5\underset{\sim}{j}$ and $\underset{\sim}{b} = -3\underset{\sim}{i} - \underset{\sim}{j}$.

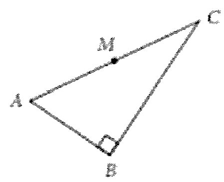
(a) Find the scalar projection of $\underset{\sim}{a}$ in the direction of $\underset{\sim}{b}$.

(b) Find the vector projection of $\underset{\sim}{a}$ onto $\underset{\sim}{b}$.

(c) Find the vector projection of $\underset{\sim}{a}$ perpendicular to the direction of $\underset{\sim}{b}$.

(d) Hence, express the vector $\underset{\sim}{a} = 2\underset{\sim}{i} - 5\underset{\sim}{j}$ in terms of projections parallel to and perpendicular to $\underset{\sim}{b} = -3\underset{\sim}{i} - \underset{\sim}{j}$.

35 ΔABC is right-angled with M being the midpoint of the hypotenuse AC, as shown.
Let $\overrightarrow{AM} = \underset{\sim}{a}$ and $\overrightarrow{BM} = \underset{\sim}{b}$.



(a) Find $\overrightarrow{AB}$ and $\overrightarrow{BC}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

(b) Prove that M is equidistant from the three vertices of ΔABC.

36 $OABC$ is a parallelogram where $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OC} = \underset{\sim}{c}$. M and N are the midpoints of $\overrightarrow{AB}$ and $\overrightarrow{BC}$ respectively.

(a) Draw a diagram of parallelogram $OABC$, showing the given vectors and midpoints.

(b) Find the vectors $\overrightarrow{OM}$ and $\overrightarrow{ON}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{c}$ and show them on your diagram.

(c) Hence find the vector $\overline{MN}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{c}$.

(d) Find vector $\overrightarrow{AC}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{c}$ and show this on your diagram.

(e) P is a point on $\overrightarrow{OM}$ such that $\overrightarrow{OP} = \frac{2}{3}\overrightarrow{OM}$. Find the vector $\overrightarrow{OP}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{c}$.

(f) Q is a point on $\overrightarrow{ON}$ such that $\overrightarrow{OQ} = \frac{2}{3}\overrightarrow{ON}$. Find the vector $\overrightarrow{OQ}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{c}$.

(g) Show that vector $\overrightarrow{MN}$ is parallel to and half the magnitude of $\overrightarrow{AC}$.

(h) Find vectors $\overrightarrow{AP}$, $\overrightarrow{PQ}$ and $\overrightarrow{QC}$, and hence prove that the diagonal $\overrightarrow{AC}$ is trisected at P and Q.