INTRODUCTION TO FUNCTIONS

1 Let $f(x) = 2x + 3$. Calculate the following function values.

a $f(1)$ b $f(0)$ c $f(-2)$ d $f(4)$

$f(1) = 2\times1+3$
$f(1) = 5$

$f(0) = 2\times0+3$
$f(0) = 3$

$f(-2) = 2\times(-2)+3$
$f(-2) = -1$

$f(4) = 2\times4+3$
$f(4) = 11$

6 Copy and complete the table of values for each function.

a $y = 2x + 1$

x	-1	0	1
y	-1	1	3

b $y = x^2 - 2x$

x	-1	0	1	2	3
y	3	0	-1	0	3

8 Given that $f(x) = x^2 - 3x + 5$, find the value of:

a $\frac{1}{2}\big(f(2) + f(3)\big)$ b $\frac{1}{4}\big(f(-1) + 2f(0) + f(1)\big) = E$

$\frac{1}{2}\left(f(2)+f(3)\right) = \frac{1}{2}\left[2^2-3\times2+5+3^2-3\times3+5\right]$

$= \frac{1}{2}\left[3+5\right] = \frac{8}{2} = 4$

$E = \frac{1}{4}\left[9+2\times5+3\right]$

$E = 22/4 = 11/2$

11 A restaurant offers a special deal to groups by charging a cover fee of \$50, then \$20 per person. Write down C, the total cost of the meal in dollars, as a function of x, the number of people in the group.

$C = 50 + 20\times x$

12 In each case explain why the function value cannot be found.

a $F(0)$, where $F(x) = \sqrt{x - 4}$. b $H(3)$, where $H(x) = \sqrt{1 - x^2}$.

when $x=0$ $x-4=-4$ which is negative
we can't calculate the square root of a negative number, so $f(0)$ cannot be calculated

when $x=3$ $1-3^2=-8<0$
Same reason.

14 Find $g(a)$, $g(-a)$ and $g(a + 1)$ for each function.

a $g(x) = 2x - 4$ b $g(x) = 2 - x$ c $g(x) = x^2$ d $g(x) = \frac{1}{x-1}$

$g(a) = 2a-4$
$g(-a) = 2(-a)-4$
$= -2a-4$
$g(a+1) = 2(a+1)-4$
$= 2a-2$

$g(a) = 2-a$
$g(-a) = 2+a$
$g(a+1) = 2-(a+1)$
$= 1-a$

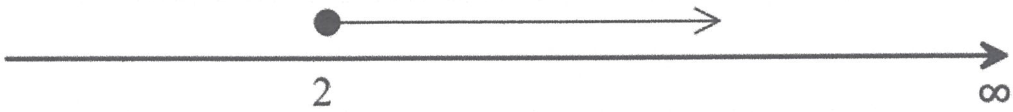
$g(a) = a^2$
$g(-a) = a^2$
$g(a+1) = (a+1)^2$
$= a^2+2a+1$

$g(a) = \frac{1}{a-1}$
$g(-a) = \frac{1}{-a-1}$
$g(a+1) = \frac{1}{a+1-1}$
$= \frac{1}{a}$

SET NOTATION & INTERVAL NOTATION

For the number line graphs below, describe the intervals in **set notation** and **interval notation**. The first one has been done for you.

1)

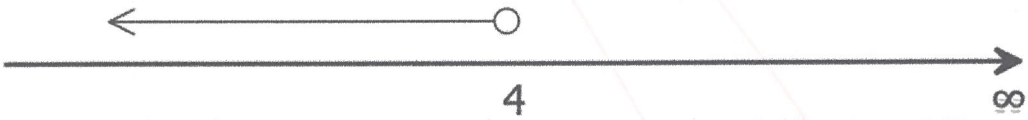


set notation: $x \geq 2$ interval notation: $x \in [2,+\infty)$

2)

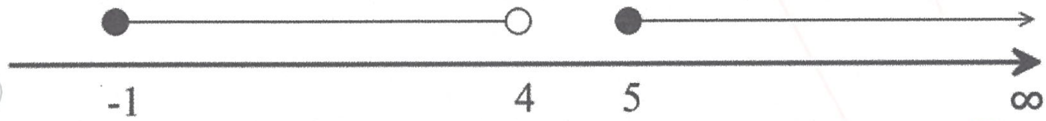
$x > -3$ $x \in (-3,+\infty)$

3)



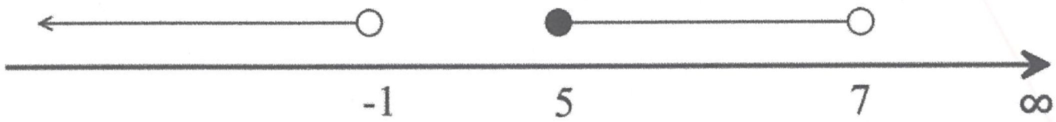
$x < 4$ $x \in (-\infty, 4)$

4)



$-1 \leq x < 4 \cup 5 \leq x$ $x \in [-1,4) \cup [5,+\infty)$

5)



$x < -1 \cup 5 \leq x < 7$ $x \in (-\infty,-1) \cup [5,7)$

DOMAIN AND RANGE

The domain of a function $y = f(x)$ is the set of all x values for which $f(x)$ is defined.

The range of a function $y = f(x)$ is the set of all y values for which $f(x)$ is defined.

Interval notation

- $[a, b]$ means the interval is between a and b, including a and b
- (a, b) means the interval is between a and b, excluding a and b
- $[a, b)$ means the interval is between a and b, including a but excluding b
- $(a, b]$ means the interval is between a and b, excluding a but including b
- $(-\infty, \infty)$ means that the interval includes the set of all real numbers R

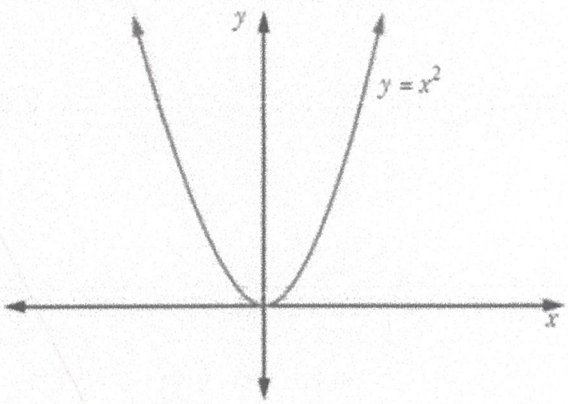
EXAMPLE 8

Find the domain and range of each function.

a $f(x) = x^2$ **b** $y = \sqrt{x-1}$

Solution

a You can find the domain and range from the equation or the graph.
For $f(x) = x^2$, you can substitute any value for x. The y values will be 0 or positive.
So the domain is all real values of x and the range is all $y \geq 0$.
We can write this using interval notation:
Domain: $(-\infty, \infty)$
Range: $[0, \infty)$



b The function $y = \sqrt{x-1}$ is only defined if $x - 1 \geq 0$ because we can only evaluate the square root of a positive number or 0.
For example, $x = 0$ gives $y = \sqrt{-1}$, which is undefined for real numbers.

So $x - 1 \geq 0$
$x \geq 1$

Domain: $[1, \infty)$
The value of $\sqrt{x-1}$ is always positive or zero. So $y \geq 0$.
Range: $[0, \infty)$

Find the natural domain and range of each function.

	Natural domain		**Range**	
	Set Notation	Interval notation	Set notation	Interval notation
$f(x) = x + 2$	$x \in \mathbb{R}$ ← "belongs to"	$x \in (-\infty, +\infty)$	$\mathbb{R}$	$(-\infty, +\infty)$
$f(x) = \sqrt{x}$	$x \in \mathbb{R}^+$	$x \in [0, +\infty)$	$\mathbb{R}^+$	$(0, +\infty)$
$f(x) = x^2$	$x \in \mathbb{R}$	$x \in (-\infty, +\infty)$	$\mathbb{R}^+$	$(0, +\infty)$
$f(x) = 2 - x^2$	$x \in \mathbb{R}$	$x \in (-\infty, +\infty)$	$x \leq 2$	$(-\infty, 2]$
$f(x) = \sqrt{x + 2}$	$x \geq -2$	$[-2, +\infty)$	$\mathbb{R}^+$	$[0, +\infty)$
$f(x) = \sqrt{3 - x}$	$x \leq 3$	$(-\infty, 3]$	$\mathbb{R}^+$	$[0, +\infty)$
$f(x) = \frac{1}{x}$	$x \neq 0$ or $\mathbb{R} - \{0\}$	$(-\infty, 0) \cup (0, +\infty)$ ↑ "union"	$\mathbb{R} - \{0\}$	$(-\infty, 0) \cup (0, +\infty)$
$g(x) = 5 - x^2$	$\mathbb{R}$	$(-\infty, +\infty)$	$x \leq 5$	$(-\infty, 5]$
$h(x) = \frac{1}{x-2}$	$x \neq 2$ or $\mathbb{R} - \{2\}$	$(-\infty, 2) \cup (2, +\infty)$	$\mathbb{R} - \{0\}$	$(-\infty, 0) \cup (0, +\infty)$
$f(x) = \sqrt{5 + x}$	$x \geq -5$	$[0, +\infty)$	$\mathbb{R}^+$	$[0, +\infty)$
$f(x) = \sqrt{16 - x^2}$	$-4 \leq x \leq 4$	$[-4, 4]$	$0 \leq x \leq 4$	$[0, 4]$
$h(x) = \frac{1}{\sqrt{2x-3}}$	$x > 3/2$	$\left(\frac{3}{2}, +\infty\right)$	$\mathbb{R}^+ - \{0\}$	$(0, +\infty)$

DEFINITION OF A FUNCTION

A function is a relationship which associates **only one value of** y to a value of x.

1 State whether each set of ordered pairs represents a function.

a (0, 2), (1, 3), (2, 4), (3, 5), (4, 6)

yes

b (1, 2), (2, 4), (3, 6), (4, 8), (5, 10)

yes.

c (2, 5), (2, –5), (3, 8), (4, –2), (5, 1)

NO as 2 has 2 images (5 and –5)

d (3, 10), (4, 9), (5, 8), (6, 7), (7, 6)

yes.

2 Which table of values represents a function?

A:

x	2	2	2	2	2
y	3	4	5	6	7

NO

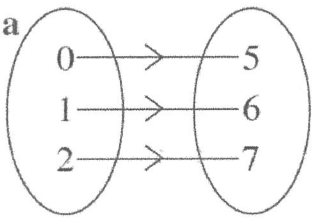
B:

x	0	1	2	3	4
y	5	5	5	5	5

Yes.

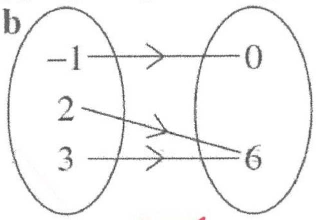
3 State whether these mapping diagrams represent functions.

a



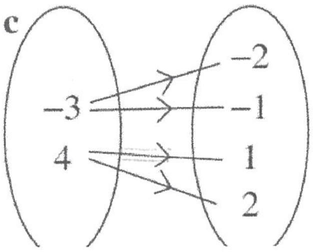
yes

b



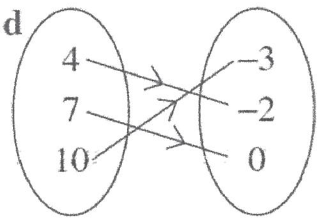
~~NO~~ yes

c



NO

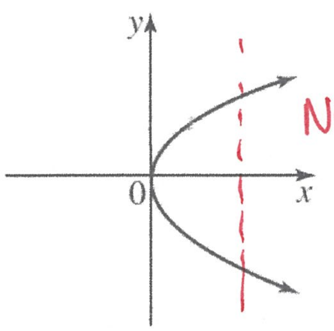
d



yes.

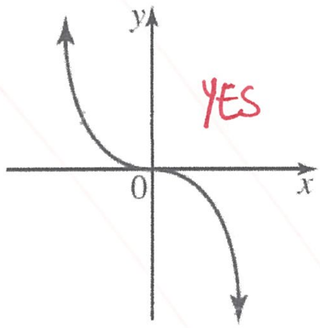
4 Use the vertical line test to determine whether each graph represents a function.

a



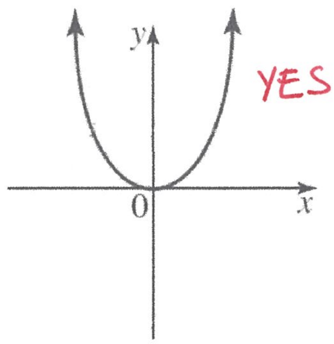
NO

b



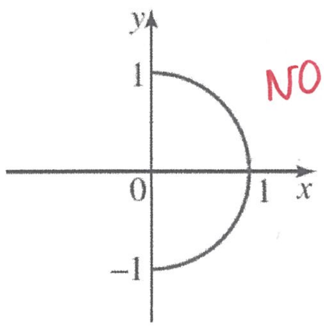
YES

c



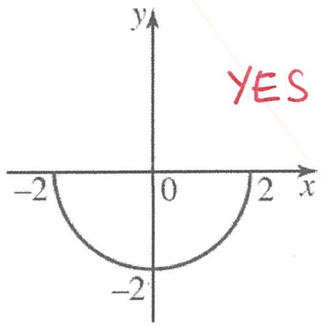
YES

d



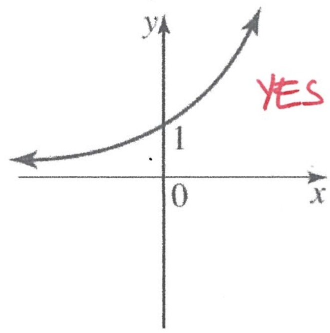
NO

e



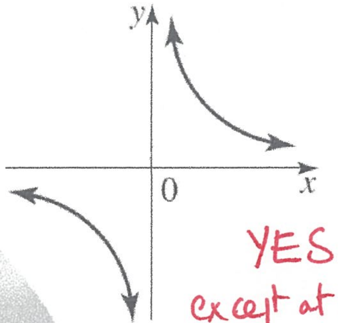
YES

f



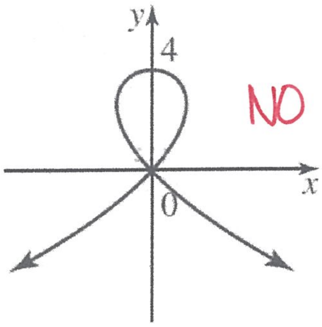
YES

g



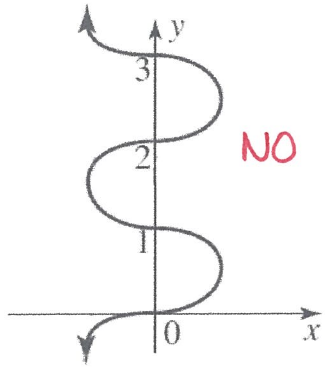
YES
except at 0

h



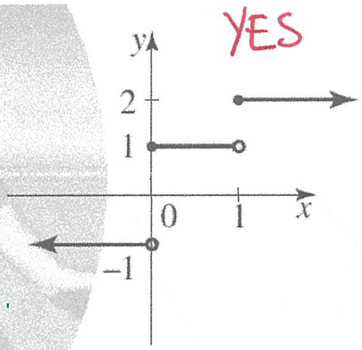
NO

i



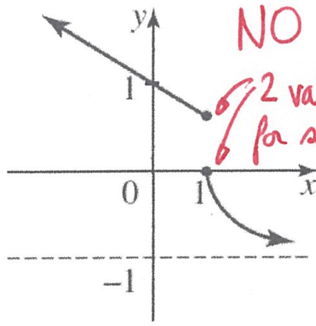
NO

j



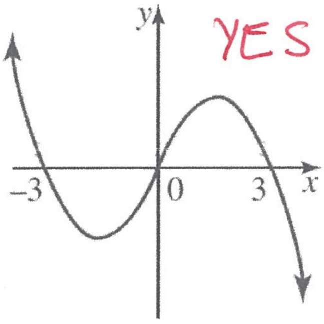
YES

k



NO
2 values for same x

l



YES

5 Explain the circumstances under which a straight line:

a must be a function **b** cannot be a function.

is a function if a vertical line cuts the straight line only at one point. So vertical lines are not functions.

6 Determine whether each of the following equations represents a function.

a $y = 2x + 3$ **b** $x = 4$ **c** $y = -2$ **d** $y = x^2$

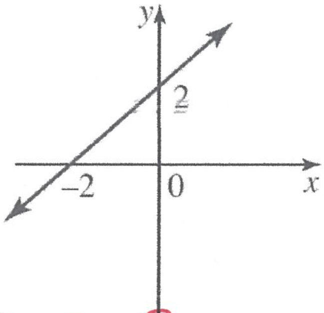
YES NO YES YES

e $y = 1 - x^2$ **f** $x^2 + y^2 = 4$ **g** $y = \frac{1}{x}$ **h** $y = \sqrt{9 - x^2}$

YES NO YES, except at 0 YES, on its natural domain

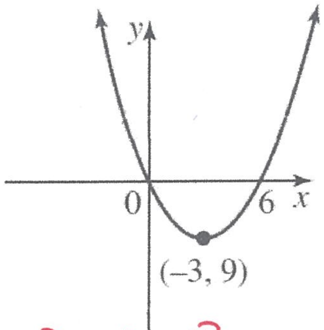
7 Find the permissible x- and y-values for each of the following.

a



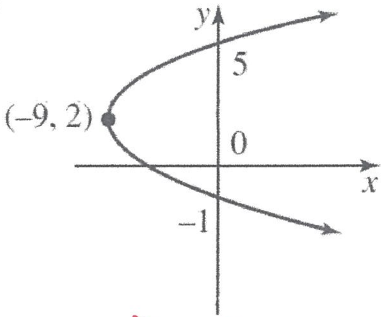
Domain $\mathbb{R}$
Range $\mathbb{R}$

b



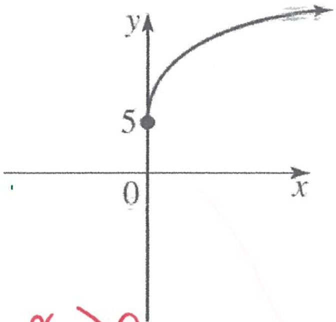
Domain $\mathbb{R}$
Range $[-3, +\infty)$

c



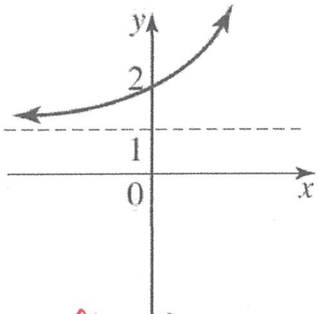
$x \geqslant -9$
y : all values possible.

d



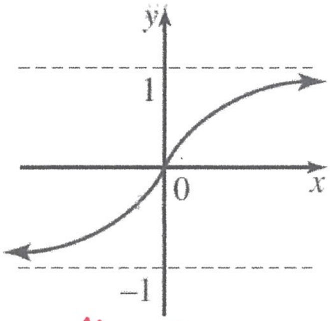
$x \geqslant 0$
$y \geqslant 5$

e



x : all values possible
$y > 1$

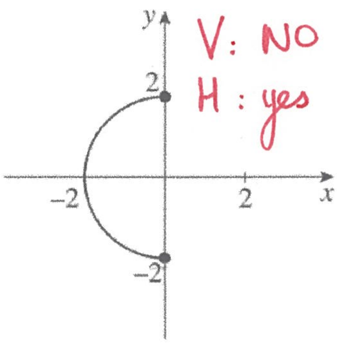
f



x : all values possible
$-1 < y < 1$

3 a Say whether each relation sketched below passes the vertical line test, and whether it passes the horizontal line test.

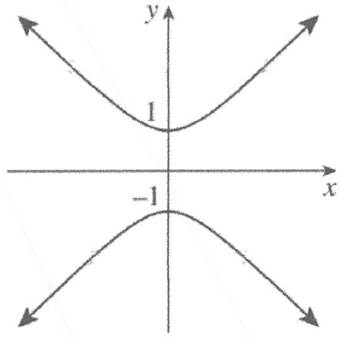
i



V: NO
H: yes

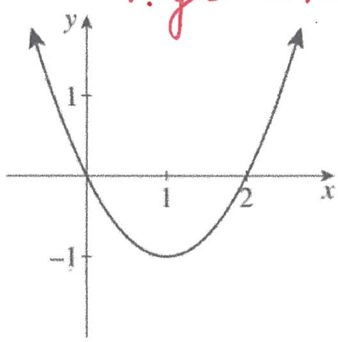
ii

V: No H: No

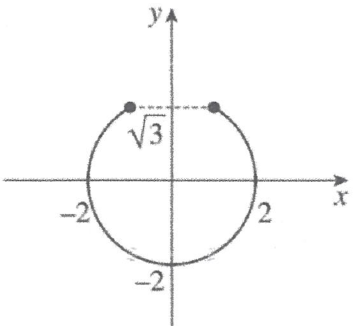


iii

V: yes H: no

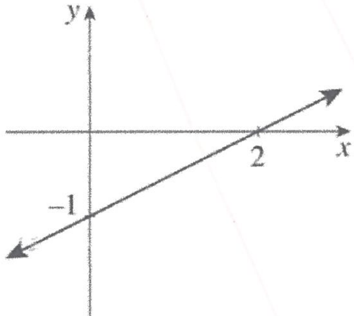


iv



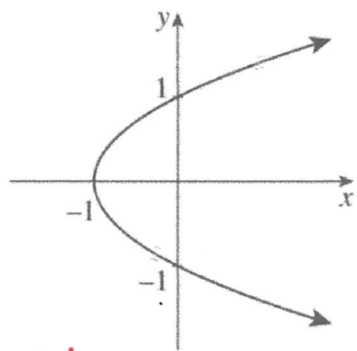
V: NO
H: No

v



V yes
H: yes

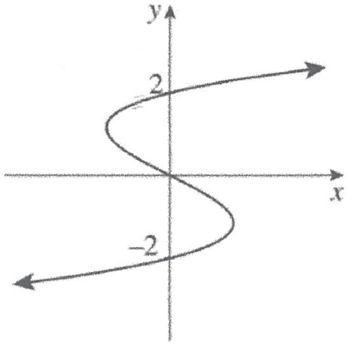
vi



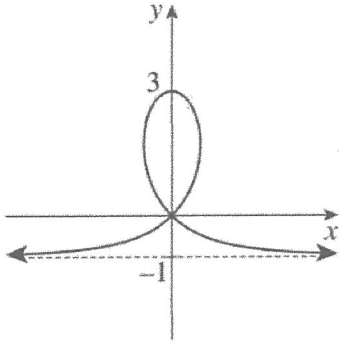
V: no
H: yes

4 Classify each graph as one-to-one, many-to-one, one-to-many or many-to-many.

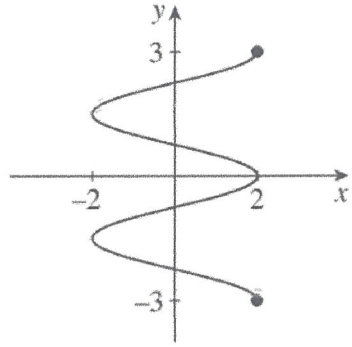
a



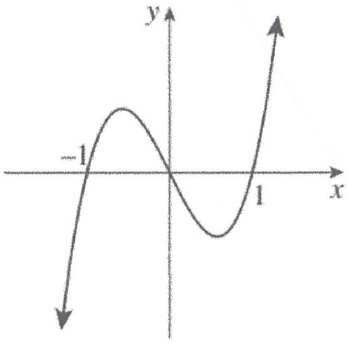
b



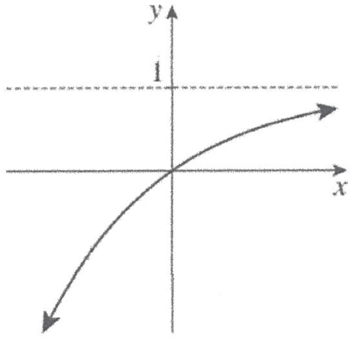
c



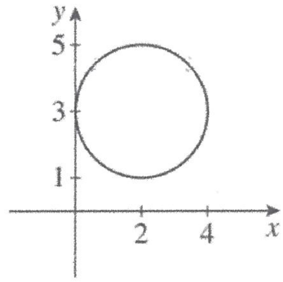
d



e



f



a) one-to-many b) many-to-many c) one-to-many

d) many-to-one e) one-to-one f) many-to-many