APPLICATIONS OF THE EXPONENTIAL AND LOGARITHMIC FUNCTIONS

2 Find the coordinates of the turning point of the curve $y = xe^{-0.5x}$ and state whether it is a maximum or minimum. Find the values of x for which:

(a) $y > 0$ **(b)** $\frac{dy}{dx} > 0$

3 Consider the function defined by the rule $f(x) = 3 - e^{-x}, x \geq 0$.

(a) Find the value of $f(0)$ and $f'(0)$.

(b) Show that $f'(x) > 0$ for all values of x in the domain.

(c) What is the value of $\lim_{x \to \infty} f(x)$?

(d) Sketch the graph of $f(x)$.

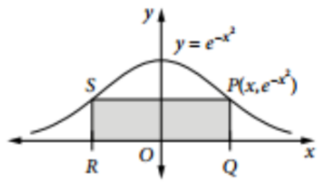
4 Consider the function defined by $f(x) = e^{-x^2}$ for all values of x.

(a) Find $f'(x)$.

(b) Find the values of x for which: **(i)** $f'(x) = 0$ **(ii)** $f'(x) > 0$ **(iii)** $f'(x) < 0$.

(c) Sketch the graph of the function.

8 The rectangle $PQRS$ has two vertices on the x-axis and two on the curve $y = e^{-x^2}$, as shown in the diagram. Find:



(a) the value of x for which the rectangle has a maximum area

(b) the maximum area of the rectangle.

9 Find the coordinates of any maximum or minimum turning points on the curve $y = \dfrac{\ln x}{x}$.

12 **(a)** Find all values of x between 0 and 2π for which $\log_e (\sin x)$ is defined.

(b) Find the maximum value of $\log_e (\sin x)$ and when it occurs.

14 When a uniform chain is suspended at two fixed points, it hangs in a catenary whose equation is $y = \frac{1}{2a}\left(e^{ax} + e^{-ax}\right)$

(a) Sketch the curve when $a = 0.5$ and the fixed points are at the same horizontal level and 8 units apart.

(b) Find the sag at the centre.

(c) Find the angle of inclination of the chain at the supports.

15 $f(x)$ is defined as $f(x) = e^{-x} \cos x$ in the domain $[0, \pi]$.

(a) Find $f(0)$, $f\left(\frac{\pi}{2}\right)$ and $f(\pi)$.

(b) Find $f'(x)$.

(c) Evaluate $f'(0)$ and $f'\left(\frac{3\pi}{4}\right)$.

(d) Sketch the graph of $y = f(x)$.