DEPENDENT EVENTS

Two events are dependent if the outcome of one event if affected by the outcome of the other event.

In that case, the probability of both events together [$P(A\ and\ B)$, also noted $P(A \cap B)$] is not the same as the probability of both events separately, i.e.:

$$P(A \cap B) \neq P(A) \times P(B)$$

For dependent events:

$$P(A \cap B) = P(A) \times P(B|A)$$

where $P(B|A)$ is the probability of B occurring given that A has already occurred.

Example 17

Two dice are rolled. A is the event '5 with the first die' and B is the event 'sum of the numbers on the two dice exceeds 10'.

(a) Find $P(A \text{ and } B)$ (also written $P(A \cap B)$).

(b) Are A and B independent?

(c) Find the probability that the sum of the numbers exceeds 10, given that the first die rolls a 5.

Solution

(a) Rolling two dice has 36 possible outcomes.

$A = \{(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)\}$, so $P(A) = \frac{6}{36} = \frac{1}{6}$

$B = \{(5, 6), (6, 5), (6, 6)\}$, so $P(B) = \frac{3}{36} = \frac{1}{12}$

A and $B = A \cap B = \{5, 6)\}$, so $P(A \text{ and } B) = \frac{1}{36}$

Note that in this case $P(A \text{ and } B) \neq P(A) \times P(B)$.

(b) Because $P(A \text{ and } B) \neq P(A) \times P(B)$, events A and B are not independent. You say they are **dependent**.

(c) You are asked to find the probability of event B given that event A has happened. This can be written as $P(B \mid A)$.

Because A has occurred, the only possible events in this case are (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6). Of these 6 outcomes, only one is favourable: (5, 6).

Hence $P(B \mid A) = \frac{1}{6}$.

DEPENDENT EVENTS

Example 18

From a set of 5 cards numbered 1, 2, 3, 4, 5, two cards are selected at random **without replacement** (i.e. without putting the first card back before taking the second). What is the probability that both cards are odd-numbered cards?

Solution

Let A be the event 'odd number first card' and B the event 'odd number second card'. You must find $P(A \text{ and } B)$. First: $P(A)=\frac{3}{5}$

If the first card is odd, then this leaves 2 odd cards in the remaining 4 cards, hence: $P(B \mid A)=\frac{2}{4}$

Thus: $P(A \text{ and } B)=P(A)\times P(B \mid A)=\frac{3}{5}\times\frac{2}{4}=\frac{3}{10}$

It may have been easier to say $P(\text{odd, odd})=\frac{3}{5}\times\frac{2}{4}=\frac{3}{10}$.

Example 20

A shoe manufacturer makes two types of shoes: sneakers and boots. Of the shoes at the factory, 60% are sneakers. If a random sample of 3 pairs of shoes is taken from the factory, find the probability that the sample is such that:

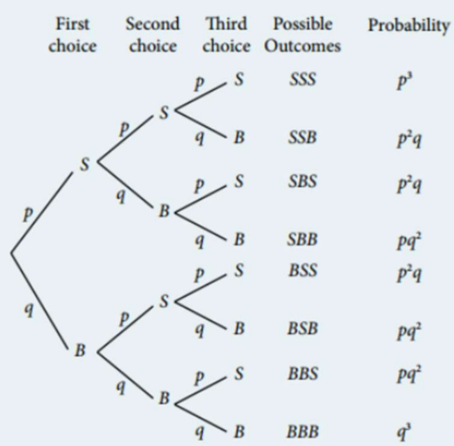
(a) there are exactly 2 pairs of sneakers

(b) there is at least 1 pair of boots

(c) the first two pairs chosen are sneakers and the third pair are boots.

Solution

Let $p=0.6$ and $q=0.4$ be the respective probabilities of choosing a pair of sneakers and a pair of boots.



(a) Required probability $= P(SSB) + P(SBS) + P(BSS)$

$= 3p^2q$

$= 3\times 0.6^2\times 0.4$

$= 0.432$

(b) Required probability $= 1 - P(0 \text{ boots})$

$= 1 - P(SSS)$

$= 1 - p^3$

$= 1 - 0.6^3$

$= 0.784$

(c) Required probability $= P(SSB)$

$= p^2q$

$= 0.6^2\times 0.4$

$= 0.144$