RATIONAL FUNCTION INEQUALITIES (x IN DENOMINATOR)

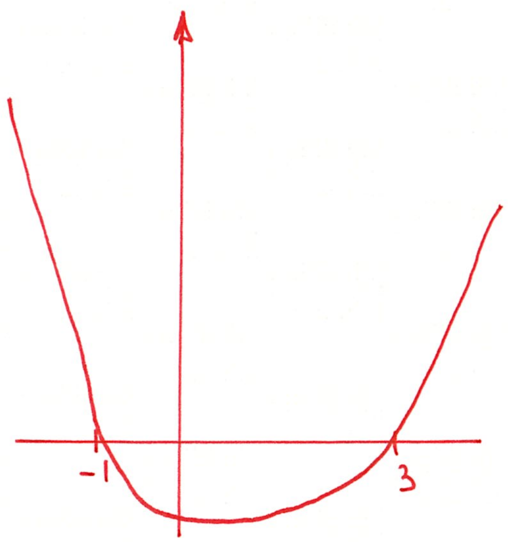
For questions **2** to **11**, solve the following inequalities.

2 $\dfrac{x-3}{x+1} > 0$

$\iff \dfrac{(x+1)^2(x-3)}{(x+1)} > 0$

$\iff (x+1)(x-3) > 0$

$f(x) = (x+1)(x-3)$ is a parabola, concave up (as the coefficient of x^2 is positive) with roots -1 and 3



So for the value $(x+1)(x-3)$ to be positive, we must have $x > 3$ or $x < -1$

3 $\dfrac{x-2}{x+3} > -2$

$\iff \dfrac{(x-2)(x+3)^2}{(x+3)} > -2(x+3)^2$

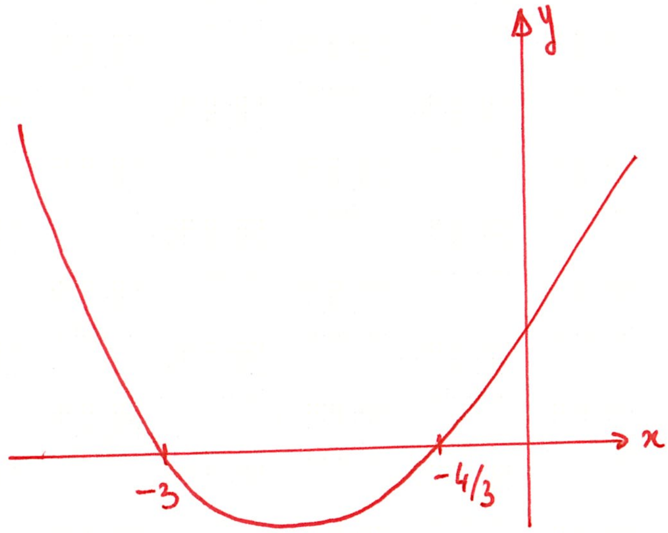
$\iff (x-2)(x+3) > -2(x+3)^2$

$\iff (x-2)(x+3) + 2(x+3)^2 > 0$

$\iff (x+3)\left[(x-2) + 2(x+3)\right] > 0$

$\iff (x+3)\left[3x+4\right] > 0$

two roots -3 and $-4/3$



So we must have either $x < -3$ or $x > -4/3$

RATIONAL FUNCTION INEQUALITIES (x IN DENOMINATOR)

4 $\dfrac{4x-3}{2x+1} \leq 3$

$\Leftrightarrow \dfrac{(4x-3)(2x+1)^2}{(2x+1)} \leq 3(2x+1)^2$

$\Leftrightarrow (4x-3)(2x+1) \leq 3(2x+1)^2$

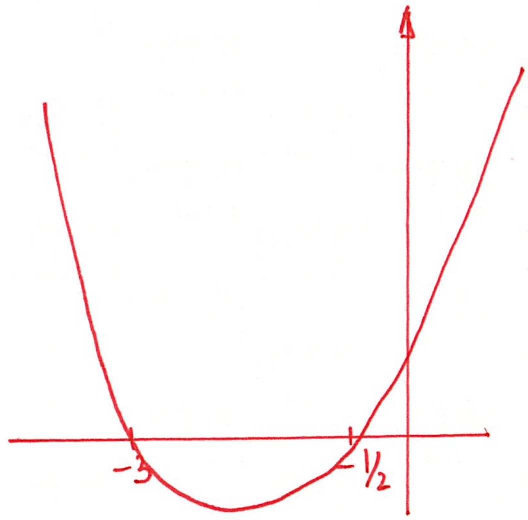
$\Leftrightarrow (4x-3)(2x+1) - 3(2x+1)^2 \leq 0$

$\Leftrightarrow (2x+1)\left[(4x+3) - 3(2x+1)\right] \leq 0$

$\Leftrightarrow (2x+1)\left[-2x-6\right] \leq 0$

$\Leftrightarrow (2x+1)(x+3) \geq 0$

two roots $-\frac{1}{2}$ and -3



So $x \leq -3$

and $x \geq -\frac{1}{2}$

But we need to exclude $-\frac{1}{2}$

otherwise $2x+1 = 0$

So $x > -\frac{1}{2}$ and $x \leq -3$

5 $\dfrac{2}{1-x} > -1$

$\Leftrightarrow \dfrac{2(1-x)^2}{(1-x)} > -1(1-x)^2$

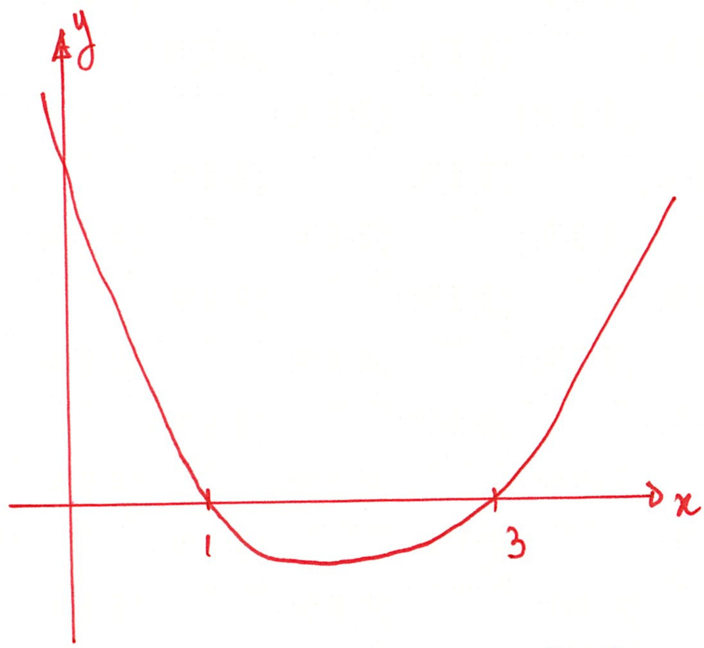
$\Leftrightarrow 2(1-x) > -(1-x)^2$

$\Leftrightarrow 2(1-x) + (1-x)^2 > 0$

$\Leftrightarrow (1-x)\left[2 + (1-x)\right] > 0$

$\Leftrightarrow (1-x)(3-x) > 0$

two roots 1 and 3



$x < 1$ and $x > 3$

RATIONAL FUNCTION INEQUALITIES (x IN DENOMINATOR)

6 $\dfrac{2x-3}{4x-5}+2<0$

$$\Leftrightarrow \frac{(2x-3)(4x-5)^2}{(4x-5)} + 2(4x-5)^2 < 0$$

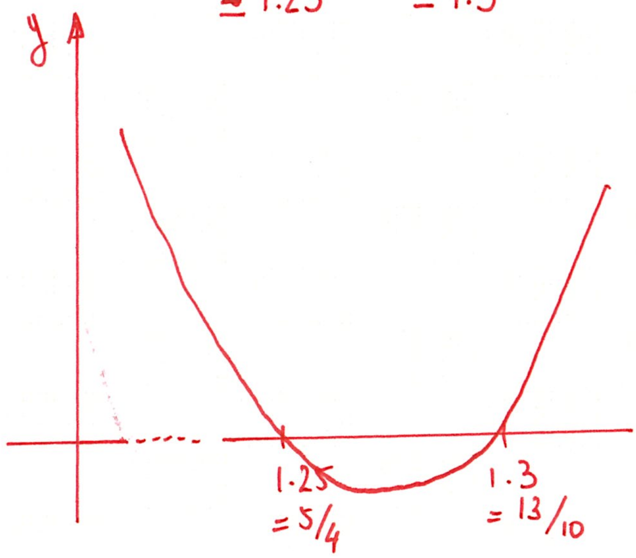
$$\Leftrightarrow (2x-3)(4x-5) + 2(4x-5)^2 < 0$$

$$\Leftrightarrow (4x-5)\left[(2x-3)+2(4x-5)\right] < 0$$

$$\Leftrightarrow (4x-5)\left[10x-13\right] < 0$$

two roots $5/4$ and $13/10$

$\simeq 1.25$ $= 1.3$



So we must have

$$\frac{5}{4} < x < \frac{13}{10}$$

as the interval solution

7 $\dfrac{1}{(x-1)(x-3)} \le -1$

$$\Leftrightarrow \frac{(x-1)^2(x-3)^2}{(x-1)(x-3)} \le -1(x-1)^2(x-3)^2$$

$$\Leftrightarrow (x-1)(x-3) \le -(x-1)^2(x-3)^2$$

$$\Leftrightarrow (x-1)(x-3) + (x-1)^2(x-3)^2 \le 0$$

$$\Leftrightarrow (x-1)(x-3)\left[1+(x-1)(x-3)\right] \le 0$$

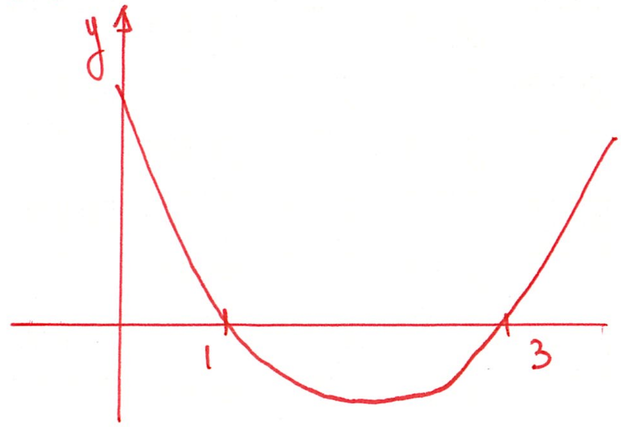
$$\Leftrightarrow (x-1)(x-3)\left[x^2-4x+4\right] \le 0$$

$$\Leftrightarrow (x-1)(x-3)(x-2)^2 \le 0$$

$$\Leftrightarrow (x-1)(x-3) \le 0$$

(we divided both sides by $(x-2)^2$ which is always positive, therefore the direction of the inequality doesn't change)

two roots 1 and 3



So the interval solution is

$$1 < x < 3$$

RATIONAL FUNCTION INEQUALITIES (x IN DENOMINATOR)

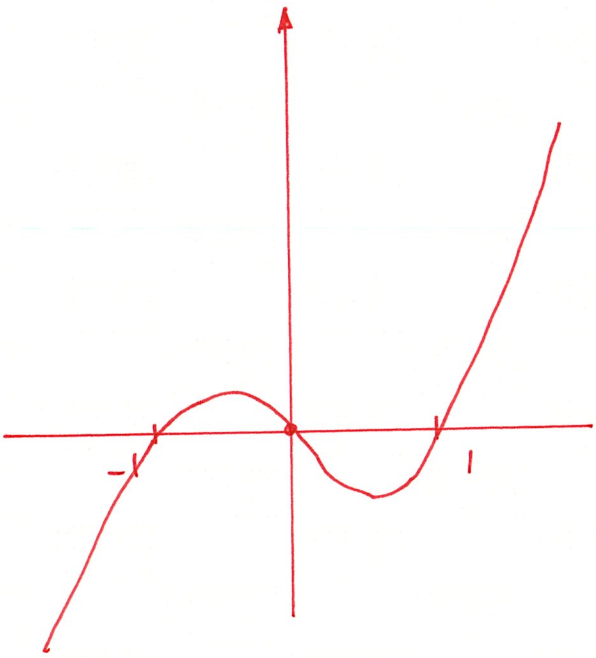
9 $\dfrac{x}{x^2-1}<0$

$$\iff \frac{x}{(x-1)(x+1)} < 0$$

$$\iff \frac{x(x-1)^2(x+1)^2}{(x-1)(x+1)} < 0$$

$$\iff x(x-1)(x+1) < 0$$

3 roots $0, -1, 1$



So the interval solutions are

$x<-1$ and $0<x<1$

10 $\dfrac{2x-4}{x+3} > \dfrac{x+2}{2x+6}$

$$\iff \frac{(2x-4)(x+3)^2}{(x+3)} > \frac{(x+2)(x+3)^2}{2(x+3)}$$

$$\iff (2x-4)(x+3) > \frac{(x+2)(x+3)}{2}$$

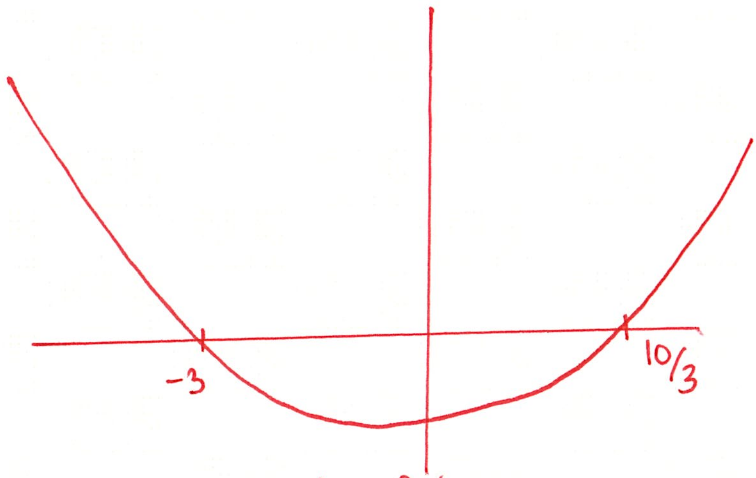
$$\iff 2(2x-4)(x+3) > (x+2)(x+3)$$

$$\iff 4(x-2)(x+3) - (x+2)(x+3) > 0$$

$$\iff (x+3)\left[4(x-2) - (x+2)\right] > 0$$

$$\iff (x+3)\left[3x-10\right] > 0$$

two roots -3 and $10/3$



So the intervals solutions are

$x<-3$ and $x>10/3$