OTHER INDUCTION QUESTIONS

Use mathematical induction to prove the following results.

1 $n^2 - 11n + 30 \geq 0$ for all integers $n \geq 6$.

2 $n^2 > -5n + 14$ for all integers $n > 2$.

OTHER INDUCTION QUESTIONS

8 **(a)** Prove that $\frac{d}{dx}\left(x^n\right) = nx^{n-1}$ for any positive integer n by:

(i) first proving S(1) that $\frac{d}{dx}(x) = 1$

(ii) then writing $x^{n+1} = x \times x^n$ and using the product rule to prove that $S(k+1)$ is true.

(b) Summarise your results to give the proof of the result by induction.

OTHER INDUCTION QUESTIONS

14 Prove that $\frac{d^n}{dx^n}\left(x^n\right) = n!$ for integral n, $n \geq 0$.

OTHER INDUCTION QUESTIONS

15 The binomial theorem states that if n is an integer, $n \geq 1$, then $(x+a)^n = \sum_{r=0}^{n} {}^nC_r x^r a^{n-r}$. Use mathematical induction to prove this result.

OTHER INDUCTION QUESTIONS

17 Prove that the number of diagonals of a convex polygon with n vertices is $\frac{n(n-3)}{2}$ for $n \geq 4$.

OTHER INDUCTION QUESTIONS

23 **(a)** Write the binomial expansion of $(k + 1)^p$ where p is a positive integer.

(b) If p is a prime number, identify which of the terms in the expansion do not have a factor of p.

(c) Prove by induction on n that if n is a positive integer and p is a prime number, then $n^p - n$ is a multiple of p.