APPLICATIONS TO PHYSICAL SITUATIONS (from CAMBRIDGE)

In this exercise take $g = 9.8\ \text{m/s}^2$.

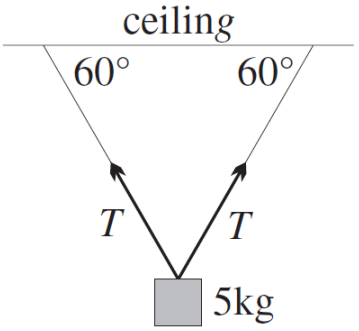
1 A ball is thrown at an angle of 30° to the horizontal with an initial speed of 20 m/s. Find the initial horizontal and vertical components of the velocity of the ball.

2 A particle has initial position vector $(4\underset{\sim}{i} + 5\underset{\sim}{j})$ metres. It moves with a constant velocity of $(3\underset{\sim}{i} - 2\underset{\sim}{j})$ m/s. Find its position vector after 7 seconds.

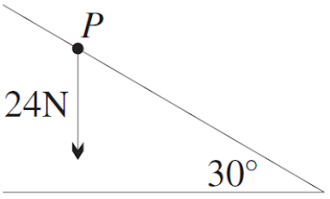
3 Find the magnitude of the resultant of the forces $(2\underset{\sim}{i} - 3\underset{\sim}{j})$ N, $(4\underset{\sim}{i} + \underset{\sim}{j})$ N and $(-3\underset{\sim}{i} + 3\underset{\sim}{j})$ N.

4 Two forces of magnitude 30 N and 16 N act away from a point P and are perpendicular. Find the magnitude and direction of the resultant force (measured from the 30 N force correct to the nearest degree).

6 In the diagram, an object of mass 5 kg is suspended from a horizontal ceiling by two strings of equal length. Each string makes an angle of 60° with the ceiling. Calculate, correct to 3 significant figures, the equal tensions in the two strings.



7 The diagram shows an object of weight 24 N at rest at P on an inclined plane. Find the component of the weight:



a down the plane,

b perpendicular to the plane.

10 A river is flowing at a speed of 1.5 m/s. Sam wants to row from point A on one bank to point B on the other bank directly opposite A. He intends to maintain a constant speed of 2.5 m/s. In what direction, correct to the nearest degree, should Sam row? Give your answer as an angle of inclination to the line AB.

11 Two dogs Brutus and Nitro are simultaneously tugging on a bone. Brutus is pulling with a force of 12 N in a direction 45° west of north, while Nitro is pulling with a force of 16 N in a direction 30° south of east. Calculate, correct to two significant figures, the magnitude and direction of the resultant force.

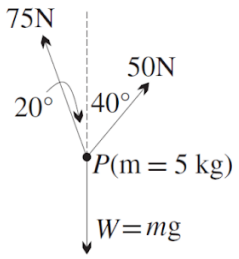
12 Three forces act on an object of mass 5 kg. These forces are represented by the vectors $9\underset{\sim}{i} - 2\underset{\sim}{j}$, $-3\underset{\sim}{i} + 10\underset{\sim}{j}$ and $18\underset{\sim}{i} - \underset{\sim}{j}$. Calculate the magnitude and direction of the acceleration of the object.

13 The position of a plane flying horizontally in a straight line at a constant speed is plotted on a radar screen. One unit on the screen represents 1 km in the air. At 12 noon the position vector of the plane is $40\underset{\sim}{i} + 16\underset{\sim}{j}$. Five minutes later its position vector is $33\underset{\sim}{i} + 40\underset{\sim}{j}$. Find:

a the position vector of the plane at 12:15 pm,

b the velocity of the plane as a vector in km/h.

14 The diagram shows an object of mass 5 kg being raised by forces of magnitude 75 N and 50 N.

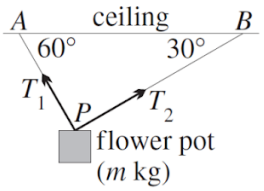


a Find the weight of the object.

b Find, correct to the nearest newton, the magnitude of the resultant of the three forces acting on the object.

c Find, correct to the nearest degree, the angle this resultant makes with the upward vertical direction.

16 In the diagram, a flowerpot of mass m kg is hung from a ceiling by two chains. Let the tensions in the chains AP and BP be T_1 and T_2 newtons respectively. The third force acting at P is the weight of the flowerpot.

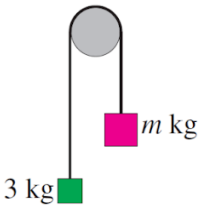


a By finding the horizontal component of the resultant of the three forces acting at P, show that $T_1 = \sqrt{3}\,T_2$.

b By finding the vertical component of the resultant of the three forces acting at P, show that $\sqrt{3}\,T_1 + T_2 = 19.6m$ newtons.

c Find the mass of the flowerpot, given that $T_2 = 98$ N.

17 The diagram shows objects of mass 3 kg and m kg attached to the ends of a light inextensible string that passes over a smooth pulley. The 3 kg object is accelerating at $4.9\ \text{m/s}^2$ upwards. Let the tension in the string be T newtons.

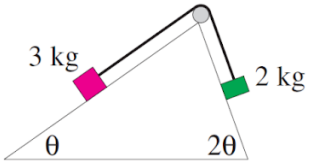


a Find the value of T.

b Find the value of m.

18 Two forces, of magnitude p newtons and q newtons, have a resultant of $2\sqrt{7}$ N when they act at 90° to each other. When they act at 30° to each other, however, the magnitude of the resultant is $2\sqrt{13}$ N. Find the values of p and q.

19 The diagram shows objects of mass 3 kg and 2 kg on connected smooth planes inclined at angles of θ and 2θ to the horizontal. The objects are attached to the ends of a light inextensible string that passes over a smooth pulley. Let T newtons be the tension in the string, and suppose that the 3 kg object is accelerating at a m/s^2 up its plane.

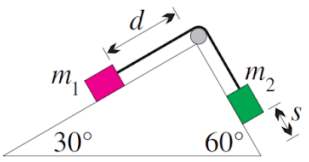


a Find, in terms of a, T, g and θ, an equation for the motion of the 3 kg object up its plane.

b Write down a similar equation for the motion of the 2 kg object down the other plane.

c Show that the system is in equilibrium when $\cos\ \theta = \frac{3}{4}$.

20 In the diagram, objects of mass m_1 and m_2 are held at rest on adjoining smooth inclined planes. They are connected by a light inextensible string that passes over a smooth pulley.



a Show that when the objects are released, the object of mass m_1 will accelerate *towards the pulley* if $m_1 < \sqrt{3}\, m_2$.

b Assuming that the condition in part **a** is satisfied, show that the object of mass m_1 will hit the pulley with speed $\sqrt{\dfrac{dg(\sqrt{3}\, m_2 - m_1)}{m_1 + m_2}}$.