THE STANDARD NORMAL DISTRIBUTION

1 If $X \sim N(15, 9)$, find the exact z values corresponding to the following x values:

(a) 18 **(b)** 21 **(c)** 22 **(d)** 16

a) $z = \dfrac{x-\mu}{\sigma} = \dfrac{18-15}{\sqrt{9}} = 1$

b) $z = \dfrac{21-15}{3} = 2$

c) $z = \dfrac{22-15}{3} = 7/3$

d) $z = \dfrac{16-15}{3} = \dfrac{1}{3}$

2 Felipe has been applying for scholarships. On one particular test that followed the distribution $X \sim N(40, 4)$, he obtained a 42; and on another test that followed the distribution $\sim N(75, 25)$, he obtained an 82.

(a) Find the z value for the first test.
(b) Find the z value for the second test.
(c) On which test did Felipe do better?

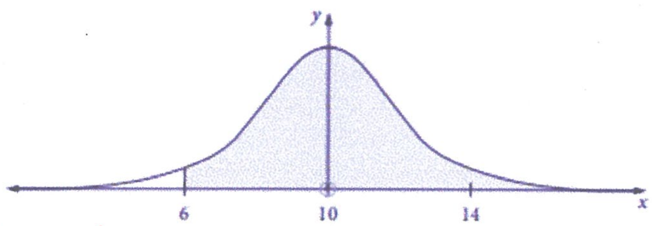
a) $z_1 = \dfrac{42-40}{2} = 1$

b) $z_2 = \dfrac{82-75}{5} = \dfrac{7}{5} = 1.4$

c) Felipe did better on the second test.

3 A normal distribution graph is shown.
If $P(X > 14) = 0.35$ then $P(X > 6)$ is equal to:

A 0.35 **B** 0.55
$\boxed{\textbf{C}}$ 0.65 **D** 0.7



$P(X>6) = P(6<X<14) + P(X>14)$

$= 2 \times P(10<X<14) + 0.35$

$= 2 \times [0.5 - P(X>14)] + 0.35$

$= 2 \times [0.5 - 0.35] + 0.35 = 2 \times 0.15 + 0.35 = 0.65$

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4 For a particular normal distribution you know that $P(X < a) = 0.214$ and $P(X < b) = 0.496$, where $a < b$.

Find the following probabilities.

(a) $P(X > a)$ **(b)** $P(X > b)$ **(c)** $P(a < X < b)$

a) $P(X>a) = 1 - P(X<a) = 1 - 0.214 = 0.786$

b) $P(X>b) = 1 - P(X<b) = 1 - 0.496 = 0.504$

c) $P(a<X<b) = P(X<b) - P(X<a)$

$= 0.496 - 0.214$

$= 0.282$

5 For $Z \sim N(0, 1)$ it is known that $P(Z < 0.85) = 0.8023$. Find:

(a) $P(Z > 0.85)$ **(b)** $P(Z < -0.85)$ **(c)** $P(Z > -0.85)$ **(d)** $P(-0.85 < Z < 0.85)$

a) $P(Z>0.85) = 1 - P(Z<0.85) = 1 - 0.8023 = 0.1977$

b) $P(Z<-0.85) = P(Z>0.85) = 0.1977$

c) $P(Z>-0.85) = 1 - P(Z<-0.85) = 0.8023$

d) $P(-0.85<Z<0.85) = 1 - P(Z<-0.85) - P(Z>0.85)$

$= 1 - 2 \times 0.1977$

$= 0.6046$

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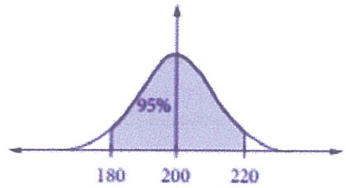
7 Given the diagram below for X which follows a normal distribution, which of the following expressions best represents the shaded area? (Z represents the standard normal distribution.)



A $P(Z < 2)$ $\boxed{\textbf{B}}$ $P(-2 < Z < 2)$
C $P(180 < Z < 220)$ **D** $P(-0.2 < Z < 0.2)$

95% of data will have z-scores between (−2) and 2.

So $P(-2 < Z < 2)$. Response $\boxed{B}$

8 X is a random variable that follows a normal distribution with a mean of 35 and a standard deviation of 7. The values of a and b are such that $P(a < X < b) = 0.95$ where this represents the middle 95% of values. a and b are best represented by:

A $a = 23.49, b = 46.51$ **B** $a = 21.28, b = 48.72$
C $a = 29.11, b = 40.89$ **D** $a = 21, b = 49$

$\mu = 35 \quad \sigma = 7$

$z = \dfrac{x - \mu}{\sigma}$ Here $P(a < X < b) = 0.95$

so $z = \pm 2$

For $z = -2$, $-2 = \dfrac{a - 35}{7}$ so $a - 35 = -14$

$a = 21$

For $z = 2$, $2 = \dfrac{b - 35}{7}$ so $b = 14 + 35 = 49$

Response $\boxed{D}$

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9 The distribution of weights for all 80.5 cm-tall girls in a population is such that the mean weight is 10.3 kg with a standard deviation of 0.8 kg.

$\mu = 10.3 \quad \sigma = 0.8$

(a) Adah is 80.5 cm tall and weighs 8.3 kg. What is Adah's z value for weight?

These z values are used as a definition for some forms of malnutrition. Moderate acute protein-energy malnutrition is defined as having a z value in the range $[-3.0, -2.0)$ and severe acute protein-energy malnutrition is defined as having a z value less than -3.0.

(b) Based on these definitions, what sort of acute protein-energy malnutrition would Adah be diagnosed with?

(c) Jamilah, who is also 80.5 cm tall, has been diagnosed with severe acute protein-energy malnutrition. What is Jamilah's weight, correct to one decimal place, less than?

(d) Another girl who is 80.5 cm tall, Xhosa, is not diagnosed with either form of acute protein-energy malnutrition. What is Xhosa's minimum weight?

a) $z = \dfrac{8.3 - 10.3}{0.8} = -2.5$

b) moderate acute protein-energy malnutrition.

c) if $z = -3$ then $\dfrac{x - 10.3}{0.8} = -3$

so $x = -3 \times 0.8 + 10.3 = 7.9$

Her weight would be ~~la~~ than 7.9 kg.

d) if $z = -2$ then $-2 = \dfrac{x - 10.3}{0.8}$

so $x = 10.3 - 2 \times 0.8 = 8.7$ kg.

So her weight would be at least 8.7 kg.

10 The percentages obtained by a group of students in a Mathematics examination are represented by a random variable M and are normally distributed with a mean of 72 and a variance of 121. All percentages are rounded to the nearest whole percentage. $\mu = 72$ $\sigma^2 = 121$ so $\sigma = 11$

(a) Calculate the probability that a student obtained a mark of at least 50% (when rounded to the nearest whole percentage) in this examination, correct to four decimal places, and the number of standard deviations that this mark is below the mean.

(b) Determine the z-score of a student who obtained a mark of 45%. What is the expected mark of a student whose z-score has the same size but opposite sign from the student who scored 45%?

To obtain an A^{++} mark, a student has to be in the top 2.5% of the group of students who have undertaken this examination.

(c) Calculate the minimum mark a student should obtain in this examination to be awarded an A^{++} by first finding the corresponding z value.

The marks in the previous year's Mathematics examination were normally distributed with a mean of 70 and a variance of 144.

(d) Would a student who obtained a mark of 94% have been awarded an A^{++} grade? Use appropriate calculations in your explanation.

a) $z = \dfrac{50-72}{11} = \dfrac{-22}{11} = -2$

But $P(-2 < Z < 2) = 95\%$ so $P(Z > -2) = 95\% + 2.5\%$
$= 97.5\%$

b) $z = \dfrac{45-72}{11} = -\dfrac{27}{11} \approx -2.45$

if $z = 2.45$ then $2.45 = \dfrac{x-72}{11}$ so $x = 11 \times 2.45 + 72$
$x = 98.95$

c) To be in the top 2.5%, you need to be above
$\mu + 2\sigma = 72 + 2 \times 11 = 94$ is the mark required

d) if $\mu = 70$ and $\sigma^2 = 144$ (i.e. $\sigma = 12$).
then $\mu + 2\sigma = 70 + 2 \times 12 = 94$
So a mark of 94 would also have been awarded an A^{++}

THE STANDARD NORMAL DISTRIBUTION

11 Anita's daily charges for gas usage in her home form a normal distribution with an average daily cost of \$7.65 and a variance of 1.44, where the random variable C represents the daily cost for the gas used.

(a) What is the probability that in any one day Anita's cost is more than \$6.45?

(b) Determine the number of standard deviations from the mean for a cost of \$8.05 and a cost of \$6.65.

(c) Plot the two z values from part **(b)** on the normal distribution curve $N(7.65, 1.2^2)$.

$N(7.65, 1.44)$ so $\mu = 7.65$ $\sigma^2 = 1.44$ so $\sigma = 1.2$

a) $P(C > 6.45)$

$6.45 = 7.65 - 1.2 = \mu - \sigma$

So $P(C > 6.45) = P(C > \mu - \sigma)$

68% of data will have z-scores between -1 and $+1$

So $P(C > 6.45) = 0.5 + 0.34 = 0.84$ 84%

b) $z = \dfrac{8.05 - 7.65}{1.2} = \dfrac{0.4}{1.2} = \dfrac{1}{3}$ so $\frac{1}{3}$ SD above the mean

$\dfrac{6.65 - 7.65}{1.2} = -\dfrac{5}{6}$ so $+\frac{5}{6}$ SD below the mean

c) $f(x) = \dfrac{1}{\sqrt{2\pi}} e^{-x^2/2}$ so $f(0) = \dfrac{1}{\sqrt{2\pi}} \sim 0.4$