SOLVING DIFFERENTIAL EQUATIONS OF THE FORM *dy/dx = f(x)*

1 Write the general solution of the following differential equations.

(a) $\frac{dy}{dx} = 2x - 1$ **(b)** $f'(x) = x^2\sqrt{x}$ **(c)** $y'(x) = 2\cos 2x$ **(d)** $y'(x) = 2\cos^2 x$

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1 Write the general solution of the following differential equations.

(e) $\frac{dz}{dt} = \frac{1}{t^2+4}$ (f) $\frac{dz}{dt} = \frac{t}{t^2+4}$ (g) $\frac{dx}{d\theta} = \sin^2\theta + \cos^2\theta$ (h) $f'(x) = 1 - e^{\frac{-x}{2}}$

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2 Find the particular solution of the following differential equations.

(a) $\frac{dy}{dx} = 2x^3 - x + 1$, given that $y = 2$ where $x = 1$

(e) $\frac{dx}{d\theta} = \frac{\sin\theta}{2 + \cos\theta}$, given that $x = 1$ where $\theta = \pi$

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3 Find the particular solution of the following differential equations.

(a) $\frac{dy}{dx}=\frac{1}{\sqrt{9-x^2}}$, $y(0)=1$ **(b)** $\frac{dx}{dt}=\frac{t}{t^2+1}$, $x=1$ where $t=0$ **(c)** $\frac{dx}{dy}=\frac{y}{2y-2}$, given that $x=1$ where $y=2$

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5 **(a)** Show that $\frac{d}{dx}\left(xe^{x}\right)=e^{x}+xe^{x}$. **(b)** Hence find $\int xe^{x}\,dx$.

(c) Find the particular solution of the differential equation $\frac{dy}{dx}=xe^{x}$, given $y(0)=-1$.

(d) Find the particular solution of the differential equation $\frac{dy}{dx}=xe^{x}-e^{x}$, given $y(0)=-2$.

(e) Hence find the particular solution for the second-order differential equation $\frac{d^{2}y}{dx^{2}}=xe^{x}$, given that $\frac{dy}{dx}=-1$ and $y=-2$ where $x=0$.

SOLVING DIFFERENTIAL EQUATIONS OF THE FORM $dy/dx = f(x)$

7 **(a)** Show that $\frac{d}{dx}\left(x + x\tan^{-1}x - \frac{1}{2}\log_e\left(x^2+1\right)\right) = \tan^{-1}x + 1$.

(b) Using **(a)**, find the particular solution of the differential equation $\frac{dy}{dx} = \tan^{-1}x + 1$ if $y(0) = 0$.

SOLVING DIFFERENTIAL EQUATIONS OF THE FORM *dy/dx = f(x)*

8 **(a)** If $\frac{dy}{dx} = \frac{1}{2}\left(e^x - e^{-x}\right)$ with initial condition $y(0) = 1$, find y.

(b) If $\frac{dz}{dx} = \frac{1}{2}\left(e^x + e^{-x}\right)$ with initial condition $z(0) = 0$, find z.

(c) Hence show that if $\frac{d^2y}{dx^2} = \frac{1}{2}\left(e^x - e^{-x}\right)$ with $y(0) = 0$ and $y'(0) = 1$, then $y = \frac{1}{2}\left(e^x - e^{-x}\right)$ is a particular solution of this equation.

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10 An oil tanker hits a reef and spills oil into the sea. The oil spills from the tanker at a rate of $\frac{10^6 t}{t^4+16}$ litres/day, where t is the number of days since the tanker first hit the reef.

It is known that $\int \frac{t}{t^4+16}dt = \frac{1}{8}\arctan\left(\frac{t^2}{4}\right)+C$.

(a) If V litres is the volume of oil spilled into the sea in the first T days, find V in terms of T.

The local newspaper report stated, 'It is expected that eventually 300 000 litres of oil will spill into the sea.'

(b) Determine whether the newspaper report is in agreement with the model above.