PROBLEMS INVOLVING FORCES

Forces exist everywhere and are fundamental to the structure of the universe and to the nature of matter.

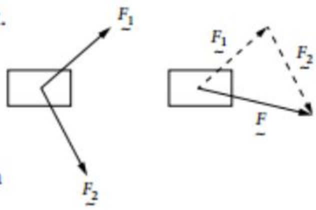
A **force** can be thought of as a push or a pull acting on an object. For most simple objects, it is reasonable to consider the forces to be acting through a single point at the object's centre of mass. (This means the object is being considered as a **particle**, as defined earlier in Chapter 7, section 7.2 Velocity and acceleration as rates of change.)

The action of a force can affect an object by changing the speed or direction of its motion, or by deforming the object. The amount of force acting on an object is measured using the SI standard unit called the newton (N). A force of 1 newton will accelerate an object of mass 1 kilogram at a rate of 1 metre per second, so $1\,\text{N} = 1\,\text{kg}\,\text{m}\,\text{s}^{-2}$.

Every object near the Earth's surface is subject to a force called gravity. This force is called the **weight** of the object and it acts vertically downwards on every object, towards the centre of mass of the Earth. The force due to gravity (the weight force) of a body of mass 1 kilogram is 9.8 N. For example, a person whose mass is 80 kg has a weight equal to their mass multiplied by g, where $g = 9.8\,\text{m}\,\text{s}^{-2}$, so an 80 kg person weighs $80 \times 9.8 = 784$ N.

Force is a vector quantity, so it needs both magnitude and direction to fully describe it. If more than one force is acting on an object, the sum of all the forces is the **resultant force** or **net force**. The object will move as if the net force is the only force that is actually acting on the object.

For example, in the diagram on the right, the resultant force acting on the object is $\underset{\sim}{F}$, the sum of the two forces $\underset{\sim}{F}_1$ and $\underset{\sim}{F}_2$ acting on the object: $\underset{\sim}{F} = \underset{\sim}{F}_1 + \underset{\sim}{F}_2$. This principle can be extended to involve the addition of many forces.



The resultant force or net force $\underset{\sim}{F}$ is the vector sum of all the forces acting on an object.

If forces $\underset{\sim}{F}_1, \underset{\sim}{F}_2, \underset{\sim}{F}_3, \ldots \underset{\sim}{F}_n$ act on an object, then $\underset{\sim}{F} = \underset{\sim}{F}_1 + \underset{\sim}{F}_2 + \underset{\sim}{F}_3 + \ldots \underset{\sim}{F}_n$

Calculating resultant forces

Example 4

Forces are acting on an object. Calculate the resultant force on the object for each of the following.

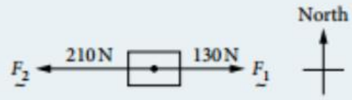
(a) $\underset{\sim}{F}_1 = 130$ N east, $\underset{\sim}{F}_2 = 210$ N west **(b)** $\underset{\sim}{F}_1 = 240$ N north, $\underset{\sim}{F}_2 = 165$ N west

Solution

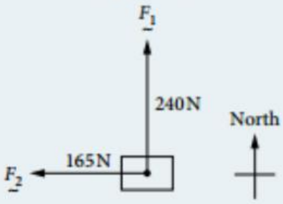
(a) Vector diagram to show the forces acting on the object:

$\underset{\sim}{F} = \underset{\sim}{F}_1 + \underset{\sim}{F}_2$

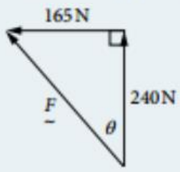
$= 130$ N east $+ 210$ N west $= 80$ N west



(b) Vector diagram to show the forces acting on the object:



Vector diagram to add the forces:



Using Pythagoras' theorem: $|\underset{\sim}{F}| = \sqrt{240^2 + 165^2}$

$= 291.25$ N

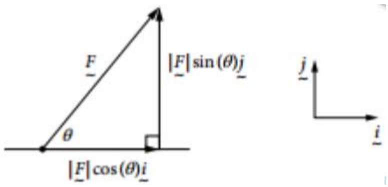
Direction of resultant force: $\tan\theta = \dfrac{165}{240}$

$\theta = \tan^{-1}\left(\dfrac{165}{240}\right)$

$\theta = 34.51°$

The resultant force is 291.25 N in the direction N34.51°W.

If the forces acting on an object are in the same plane, then each of the forces and the resultant force can be expressed in component form as a sum of two perpendicular vectors, using horizontal $(\underset{\sim}{i})$ and vertical $(\underset{\sim}{j})$ components. The force $\underset{\sim}{F}$ can be resolved into the two perpendicular components as $\underset{\sim}{F} = x\underset{\sim}{i} + y\underset{\sim}{j}$. For example, from the diagram: $\underset{\sim}{F} = |\underset{\sim}{F}|\cos\theta\underset{\sim}{i} + |\underset{\sim}{F}|\sin\theta\underset{\sim}{j}$.



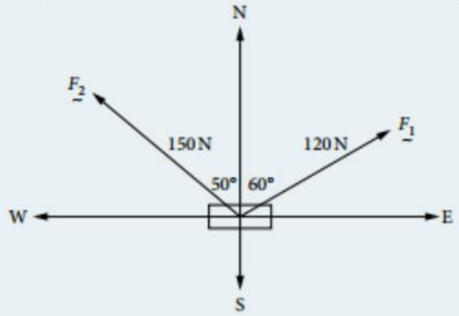
Calculating resultant forces using vector components

Example 5

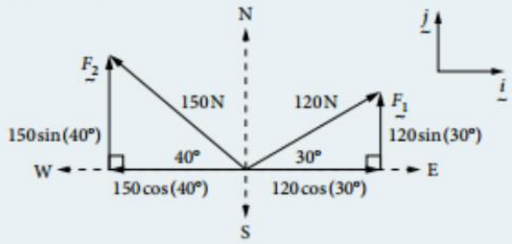
Two forces are acting on an object: $\underset{\sim}{F}_1 = 120\,\text{N}$ acting at N60°E and $\underset{\sim}{F}_2 = 150\,\text{N}$ acting at N50°W. Calculate the resultant force $\underset{\sim}{F}$ acting on the object.

Solution

Vector diagram to show the forces acting on the object:



Resolving forces into horizontal $(\underset{\sim}{i})$ and vertical $(\underset{\sim}{j})$ components:



$\underset{\sim}{F}_1 = 120\cos 30°\underset{\sim}{i} + 120\sin 30°\underset{\sim}{j}$

$= 103.92\underset{\sim}{i} + 60.00\underset{\sim}{j}$

$\underset{\sim}{F}_2 = -150\cos 40°\underset{\sim}{i} + 150\sin 40°\underset{\sim}{j}$

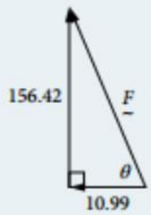
$= -114.91\underset{\sim}{i} + 96.42\underset{\sim}{j}$

$\underset{\sim}{F} = \underset{\sim}{F}_1 + \underset{\sim}{F}_2$

$= 103.92\underset{\sim}{i} + 60.00\underset{\sim}{j} - 114.91\underset{\sim}{i} + 96.42\underset{\sim}{j}$

$= (103.92 - 114.91)\underset{\sim}{i} + (60.00 + 96.42)\underset{\sim}{j}$

$= -10.98\underset{\sim}{i} + 156.42\underset{\sim}{j}$



Using Pythagoras' theorem:

$|\underset{\sim}{F}| = \sqrt{(-10.98)^2 + 156.42^2}$

$= 156.80\,\text{N}$

$\theta = \tan^{-1}\left(\dfrac{156.42}{10.98}\right)$

$= 86°$

$\therefore$ Bearing is $(270 + 86) = 356°\text{T}$

$\underset{\sim}{F} = 156.80\,\text{N}$ acting in the direction 356°T.

The method shown in Example **5**, above, can be used similarly in situations where more than two forces are acting on an object.

Calculating the resultant of three forces using vector components

Example 6

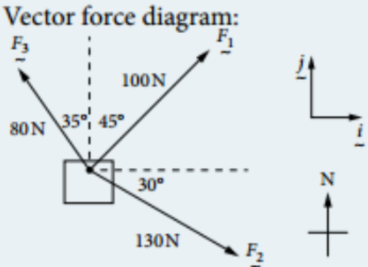
Three forces $\underset{\sim}{F}_1$, $\underset{\sim}{F}_2$ and $\underset{\sim}{F}_3$ are acting on an object: $\underset{\sim}{F}_1 = 100$ N at 045°T, $\underset{\sim}{F}_2 = 130$ N at 120°T and $\underset{\sim}{F}_3 = 80$ N at 325°T.

(a) Resolve each of the forces into horizontal $\underset{\sim}{i}$ components and vertical $\underset{\sim}{j}$ components.

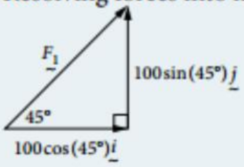
(b) Determine the resultant force $\underset{\sim}{F}$.

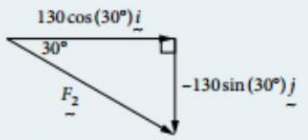
Solution

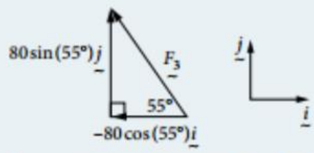
(a) Vector force diagram:



Resolving forces into horizontal $\underset{\sim}{i}$ and vertical $\underset{\sim}{j}$ components:







$$\underset{\sim}{F}_1 = 100\cos 45°\underset{\sim}{i} + 100\sin 45°\underset{\sim}{j} = 70.71\underset{\sim}{i} + 70.71\underset{\sim}{j}$$

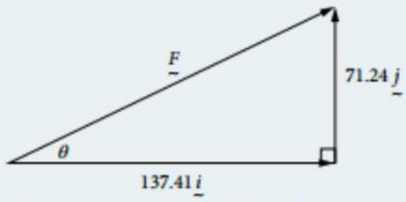
$$\underset{\sim}{F}_2 = 130\cos 30°\underset{\sim}{i} - 130\sin 30°\underset{\sim}{j} = 112.58\underset{\sim}{i} - 65.00\underset{\sim}{j}$$

$$\underset{\sim}{F}_3 = -80\cos 55°\underset{\sim}{i} + 80\sin 55°\underset{\sim}{j} = -45.89\underset{\sim}{i} + 65.53\underset{\sim}{j}$$

(b) Sum of horizontal $\underset{\sim}{i}$ and vertical $\underset{\sim}{j}$ components:

$$\underset{\sim}{F} = \underset{\sim}{F}_1 + \underset{\sim}{F}_2 + \underset{\sim}{F}_3$$
$$= (70.71 + 112.58 - 45.89)\underset{\sim}{i} + (70.71 - 65.00 + 65.53)\underset{\sim}{j}$$
$$= 137.41\underset{\sim}{i} + 71.24\underset{\sim}{j}$$

Magnitude and direction of resultant vector:



$$|\underset{\sim}{F}| = \sqrt{137.41^2 + 71.24^2} = 154.77 \text{ N}$$

$$\tan\theta = \frac{71.24}{137.41}$$

$$\theta = 27.41°$$

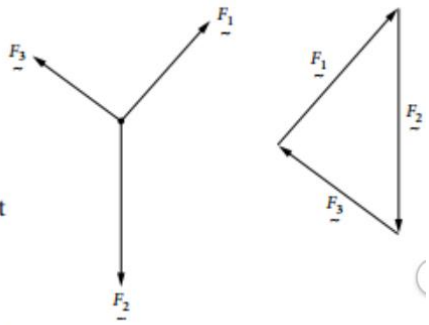
The resultant force is 154.77 N acting at an angle of 062.59°T.

Forces in equilibrium

If the resultant force acting on an object is zero, then the forces on the object are said to be in **equilibrium**. The object's motion will not change, which means that the object will remain at rest or continue with constant velocity. This is known as Newton's first law of motion.

Newton's first law of motion states that an object at rest stays at rest and an object in motion stays in motion with the same velocity, unless acted upon by an unbalanced force.

If the forces acting on an object are in equilibrium, then the resultant force or net force acting on the object is zero. For example, if the three forces $\underset{\sim}{F}_1$, $\underset{\sim}{F}_2$ and $\underset{\sim}{F}_3$ that act on a particle are in equilibrium, then $\underset{\sim}{F}_1 + \underset{\sim}{F}_2 + \underset{\sim}{F}_3 = \underset{\sim}{0}$. The forces can be displayed using a triangle of forces, as shown at right.

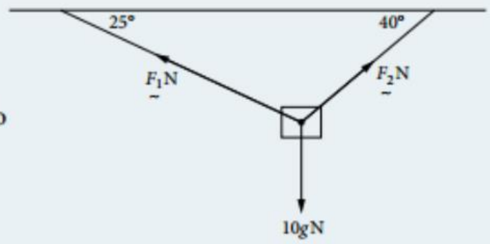


The magnitude of the forces acting and the angles between the forces can then be found using trigonometric ratios (if the triangle contains a right angle) or by using the sine or cosine rule.

In the problems in this course, strings and ropes are considered to have negligible (effectively zero) mass. A smooth, light pulley is also considered to have negligible mass and the friction between a string or rope and a pulley is considered to be negligible.

Example 7

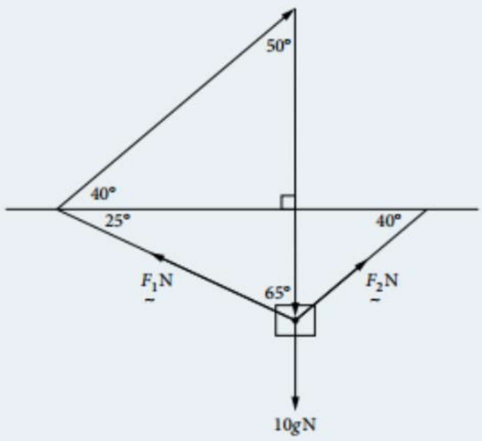
A particle of mass 10 kg is suspended by two strings attached to two points in the same horizontal plane. If the two strings make angles of 25° and 40° respectively to the horizontal, find the magnitude of the tension force in each string in newtons correct to two decimal places.

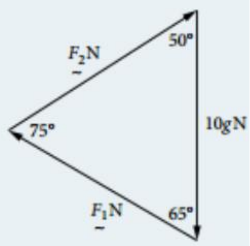


Solution

Representing the forces in a triangle:

The triangle of forces is best obtained by producing the vertical force upwards, translating $\underset{\sim}{F}_2$ to the head of $\underset{\sim}{F}_1$ and producing it until it meets the vertical line. In the original diagram the forces are not drawn to scale, so the triangle is not drawn to scale.





Using the sine rule to determine the magnitudes of the tension forces:

$$\frac{|\underset{\sim}{F}_1|}{\sin 50°} = \frac{|\underset{\sim}{F}_2|}{\sin 65°} = \frac{10g}{\sin 75°}$$

Magnitude of the first force:

$$\frac{|\underset{\sim}{F}_1|}{\sin 50°} = \frac{10g}{\sin 75°}$$

$$|\underset{\sim}{F}_1| = \frac{98}{\sin 75°} \times \sin 50°$$

$$= 77.72\text{ N}$$

Magnitude of the second force:

$$\frac{|\underset{\sim}{F}_2|}{\sin 65°} = \frac{10g}{\sin 75°}$$

$$|\underset{\sim}{F}_2| = \frac{98}{\sin 75°} \times \sin 65°$$

$$= 91.95\text{ N}$$

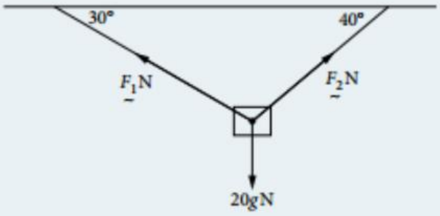
PROBLEMS INVOLVING FORCES

The magnitude of the forces acting and the angles between the forces can also be found by expressing vectors in component form. If there are not three forces acting on an object, then a triangle of forces obviously cannot be used. However, if more than one force acts on an object, then the resultant force can always be determined from adding the forces. If the object is in equilibrium, then the sum of the $\underset{\sim}{i}$ components and the sum of the $\underset{\sim}{j}$ components must each be zero.

Calculating forces in equilibrium using components

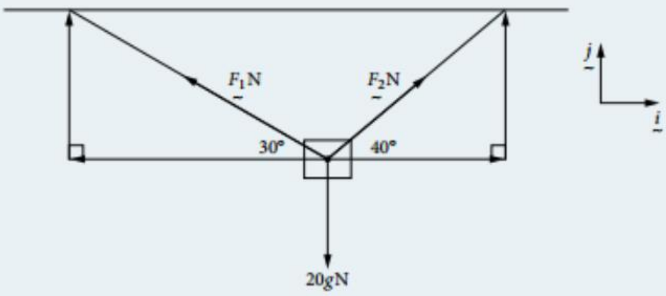
Example 8

A particle of mass 20 kg is suspended by two strings attached to two points in the same horizontal plane. If the two strings make angles of 30° and 40° respectively to the horizontal, use component form to find the magnitude of the tension force in each string, in newtons correct to one decimal place.



Solution

Resolving the forces into $\underset{\sim}{i}$ and $\underset{\sim}{j}$ components:



Vector sum in the $\underset{\sim}{i}$ direction:

$|\underset{\sim}{F}_2|\cos 40° - |\underset{\sim}{F}_1|\cos 30° = 0$

$0.7660|\underset{\sim}{F}_2| - 0.8660|\underset{\sim}{F}_1| = 0$

$|\underset{\sim}{F}_2| = 1.1305|\underset{\sim}{F}_1|$ [1]

Vector sum in the $\underset{\sim}{j}$ direction:

$|\underset{\sim}{F}_1|\sin 30° + |\underset{\sim}{F}_2|\sin 40° - 20g = 0$

$0.5000|\underset{\sim}{F}_1| + 0.6428|\underset{\sim}{F}_2| - 196 = 0$ [2]

Substituting [1] into [2] gives: $0.5000|\underset{\sim}{F}_1| + 0.6428 \times 1.1305|\underset{\sim}{F}_1| - 196 = 0$

$1.1227|\underset{\sim}{F}_1| = 196$

$|\underset{\sim}{F}_1| = 159.8\,\text{N}$

Then, from [1]: $|\underset{\sim}{F}_2| = 1.1305 \times 159.8$

$= 180.6\,\text{N}$

The tension forces in the strings have magnitudes of $|\underset{\sim}{F}_1| = 159.8$ N and $|\underset{\sim}{F}_2| = 180.6$ N respectively.