LOGARITHMS IN THE REAL WORLD

1 What is the value of $10 \log_{10}\left(\frac{P_2}{P_1}\right)$ when: **(a)** $P_2 = P_1$ **(b)** $P_2 = 100\,000 P_1$

a) if $P_2 = P_1$, then $10 \log_{10}\left(\frac{P_2}{P_1}\right) = 10 \log_{10}\left(\frac{P_1}{P_1}\right) = 10 \log_{10} 1 = 0$

b) if $P_2 = 100{,}000\, P_1$, then $10 \log_{10}\left(\frac{P_2}{P_1}\right) = 10 \log_{10}\left(\frac{100{,}000\, P_1}{P_1}\right) = 10 \log_{10}(10^5) = 50$

2 How many times louder is:

(a) a sound which is 20 dB louder than another sound

(b) a 75 dB sound than a 35 dB sound

(c) a 79 dB sound than a 72 dB sound?

a) if $20 = 10 \log_{10}\left(\frac{P_2}{P_1}\right) \Rightarrow \log_{10}\left(\frac{P_2}{P_1}\right) = 2 \Rightarrow \frac{P_2}{P_1} = 10^2 = 100$ so $P_2 = 100\, P_1$

b) $75 = 10 \log_{10}\left(\frac{P_2}{P_0}\right)$ and $35 = 10 \log_{10}\left(\frac{P_1}{P_0}\right)$

So $\underbrace{75 - 35}_{=40} = 10\left[\log_{10}\left(\frac{P_2}{P_0}\right) - \log_{10}\left(\frac{P_1}{P_0}\right)\right] = 10 \log_{10}\left(\frac{P_2}{P_1}\right)$ so $\frac{P_2}{P_1} = 10^4$, $P_2 = 10^4 \times P_1$

c) $\underbrace{79 - 72}_{=7} = 10 \log_{10}\left(\frac{P_2}{P_1}\right)$ so $\frac{P_2}{P_1} = 10^{0.7}$ so $P_2 \simeq 5\, P_1$

3 If one sound is twice as loud as another, how many more decibels is its intensity?

$10 \log_{10}\left(\frac{P_2}{P_1}\right) = 10 \log_{10}\left(\frac{2P_1}{P_1}\right) = 10 \log_{10}(2) \simeq 3$ dB

4 An earthquake measuring 8.7 on the Richter scale is followed by one that measures 6.5 on the Richter scale. How many times stronger is the first earthquake than the second?

$M_1 = \log_{10}\left(\frac{A_1}{A_0}\right) = 8.7$ and $M_2 = \log_{10}\left(\frac{A_2}{A_0}\right) = 6.5$

So $\underbrace{8.7 - 6.5}_{=2.2} = \log_{10}\left(\frac{A_1}{A_0}\right) - \log_{10}\left(\frac{A_2}{A_0}\right) = \log_{10} A_1 - \log_{10} A_0 - \log_{10} A_2 + \log_{10} A_0$

$= \log_{10}(A_1/A_2)$

So $\log_{10}(A_1/A_2) = 2.2$ so $A_1 = 10^{2.2} A_2 \simeq 158\, A_2$

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5 The energy released by an earthquake, E, can be given by $\log_{10} E = 11.8 + 1.5\, M_L$, where M_L is the measurement of its magnitude on the Richter scale. Calculate the energy released by both of the earthquakes in Question 4 and state how many times more energy is released by the first earthquake than by the second.

$\log_{10} E_1 = 11.8 + 1.5\, M_1$ so $E_1 = 10^{(11.8 + 1.5 M_1)} = 10^{(11.8 + 1.5 \times 8.7)} \simeq 7 \times 10^{24}$ J

whereas $E_2 = 10^{(11.8 + 1.5 M_2)} = 10^{(11.8 + 1.5 \times 6.5)} \simeq 3.5 \times 10^{21}$ J

So $\frac{E_1}{E_2} \simeq \frac{7 \times 10^{24}}{3.5 \times 10^{21}} \simeq 2{,}000$ approx 2000 times more energy.

There is about 2000 more times of Energy released by the earthquake of 8.7 on the Richter scale than the one of 6.5 on the Richter scale.

6 Calculate, correct to one decimal place, the pH level of a solution where the concentration of H^+ (hydrogen) ions is 2.3×10^{-5} mol/L. Is this an acidic or a basic solution?

$pH = -\log_{10}[H^+] = -\log_{10}(2.3 \times 10^{-5}) = 4.6$

which is acidic as it's less than 7.

7 The frequency of the note A3, the A below middle C, is 220 Hz. Another note has a frequency of 1760 Hz.
(a) How many octaves higher than A3 is this note? **(b)** What is this note?

a) $\log_2\left(\frac{f_2}{f_1}\right)$ is a measure of how many octaves higher a note of frequency f_2 is than a note of frequency f_1

$\log_2\left(\frac{1760}{220}\right) = \log_2 8 = \log_2 2^3 = 3$ so 3 octaves higher.

b) 220 Hz is the note A.

So exactly 3 octaves higher would also be a A note.

8 Why can you not use the decibel scale to measure a sound of zero intensity, i.e. no sound at all?

A sound of zero intensity on the decibel scale would be $10 \log_{10} 0$, which is undefined.