APPLICATIONS INVOLVING GRAPHING FUNCTIONS

Graphs of functions with algebraic denominators may have asymptotes. You also have information about turning points and points of inflection obtained by using calculus to differentiate the function. You can use all these skills to sketch a variety of functions.

Example 5

(a) Sketch the graph of $y = \frac{1}{4x-1}$.

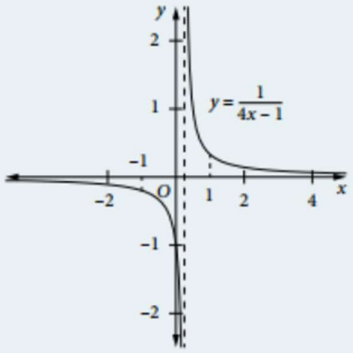
(b) Find the equation of the tangent to the curve at the point where $x = 1$.

(c) Find the equation of the normal to the curve at point where $x = -1$.

(d) Find the coordinates of the point of intersection of the tangent and normal found in parts **(b)** and **(c)**.

Solution

(a)



(b) $y = \frac{1}{4x-1}$: $\frac{dy}{dx} = -\frac{4}{(4x-1)^2}$

$x = 1$: Gradient of tangent $= -\frac{4}{9}$

$x = 1, y = \frac{1}{3}$

Equation of tangent: $y - \frac{1}{3} = -\frac{4}{9}(x-1)$

$9y - 3 = -4x + 4$

$4x + 9y - 7 = 0$

(c) $x = -1$: Gradient of tangent $= -\frac{4}{25}$

Gradient of normal $= \frac{25}{4}$

$x = -1, y = -\frac{1}{5}$

Equation of normal: $y + \frac{1}{5} = \frac{25}{4}(x+1)$

$20y + 4 = 125x + 125$

$125x - 20y + 121 = 0$

(d) $4x + 9y - 7 = 0$ [1]

$125x - 20y + 121 = 0$ [2]

[1] × 20: $80x + 180y - 140 = 0$ [3]

[2] × 9: $1125x - 180y + 1089 = 0$ [4]

[3] + [4]: $1205x + 949 = 0$

$x = -\frac{949}{1205}$

Substitute into [1]: $-\frac{3796}{1205} + 9y - 7 = 0$

$9y = \frac{12231}{1205}$

$y = \frac{1359}{1205}$

The point of intersection is $\left(-\frac{949}{1205}, \frac{1359}{1205}\right)$