OTHER EXAMPLES OF MOTIONS

1 The displacement x of a particle at time t is given by $x = 6\cos 4t + 3$. Find:

(a) the velocity and acceleration at any time t **(b)** the position of the particle when $t = 0$

(c) the values that x can take **(d)** the time when the particle first reaches the position $x = 0$.

a) $\dot{x}(t) = 24(-\sin 4t) = -24\sin 4t$ $\ddot{x}(t) = -96\cos 4t$

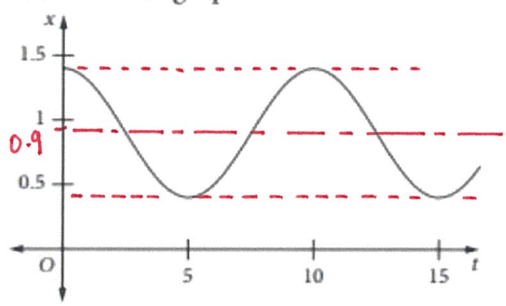
b) at $t=0$ $x(0) = 6\cos(4\times 0) + 3 = 6+3 = 9$

c) $x(t)$ varies from -3 to 9

d) $x(t) = 0$ when $6\cos 4t = -3$, i.e. $\cos 4t = -1/2$

so $4t = \frac{2\pi}{3}$ $\Rightarrow t = \frac{\pi}{6}$.

2 Consider the graph.



Which of the following functions does this graph represent?

A $x = 0.9 + \cos\frac{\pi t}{10}$ **B** $x = 0.9 + \cos\frac{\pi t}{5}$

C $x = 0.9 + 0.5\cos\frac{\pi t}{10}$ **D** $x = 0.9 + 0.5\cos\frac{\pi t}{5}$

at $t=5$ $x(5) = 0.4$ $\boxed{D}$

4 A particle moves in a straight line so that at time t its displacement from a fixed origin is x and its velocity is v. If its acceleration is $2\sin t$, and $v = 1$ and $x = 1$ when $t = 0$, find x as a function of t.

$a = 2\sin t = \ddot{x}(t)$ so $\dot{x}(t) = -2\cos t + C$

at $t=0$, $v=1$ so $-2\cos 0 + C = 1$ $\Leftrightarrow$ $C=3$

so $\dot{x}(t) = -2\cos t + 3$

then we integrate again to find $x(t)$

$x(t) = -2\sin t + 3t + K$

at $t=0$ $x=1$ so $-2\sin 0 + 3\times 0 + K = 1$ so $K=1$

$x(t) = -2\sin t + 3t + 1$

OTHER EXAMPLES OF MOTIONS

6 A particle moves in a straight line so that at time t its displacement from a fixed origin O is x, where $x = 2 + t - 2\cos t$.

(a) Write the velocity and acceleration at any time t.

(b) Find its initial displacement, velocity and acceleration.

$x(t) = 2 + t - 2\cos t$

a) $\dot{x}(t) = 1 + 2\sin t$

$\ddot{x}(t) = 2\cos t$

b) at $t = 0$ $\quad x(0) = 2 + 0 - 2\cos 0 = 2 - 2 = 0$

$\dot{x}(0) = 1 + 2\sin 0 = 1$

$\ddot{x}(0) = 2\cos 0 = 2$

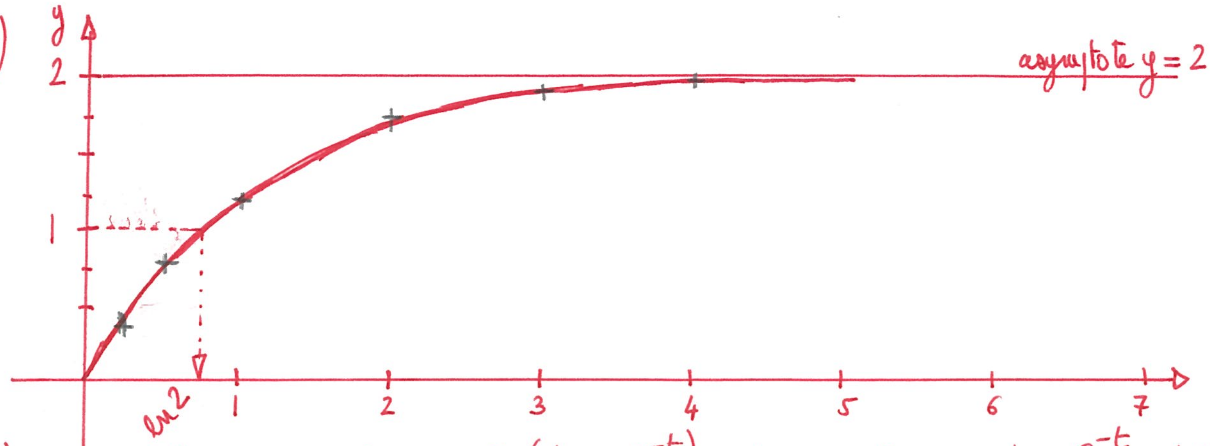
OTHER EXAMPLES OF MOTIONS

7 A particle moves in a straight line so that its displacement x from a fixed origin at any time t is given by $x(t) = 2(1 - e^{-t})$. $= 2 - 2e^{-t}$

(a) Find $x(0)$, $\dot{x}(0)$ and $\ddot{x}(0)$. **(b)** Sketch the graph of $x(t)$. **(c)** Find t when $x(t) = 1$.

$\dot{x}(t) = 2e^{-t}$ $\quad \ddot{x}(t) = -2e^{-t}$ $\quad$ a) $x(0) = 0$ $\quad \dot{x}(0) = 2$ $\quad \ddot{x}(0) = -2$

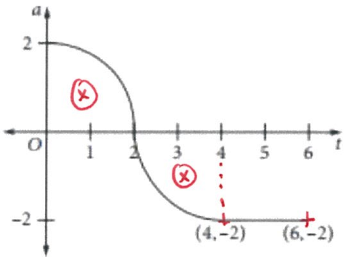
b)



c) $x(t) = 1$ when $2(1 - e^{-t}) = 1$, i.e. $1 - e^{-t} = \frac{1}{2}$

or $e^{-t} = \frac{1}{2}$ $\quad -t = \ln\left(\frac{1}{2}\right)$ $\quad$ so $t = -\ln\frac{1}{2} = \ln 2 \simeq 0.69$

9 A particle moves along the x-axis. Initially it is at rest at the origin. The graph shows the acceleration a of the particle as a function of time t for $0 \leq t \leq 6$.



(a) Write the time at which the velocity of the particle is a maximum.

(b) At what time during the interval $0 < t \leq 6$ is the particle at rest?

(c) At what time during the interval $0 \leq t \leq 6$ is the particle farthest from the origin? Give brief reasons for your answer.

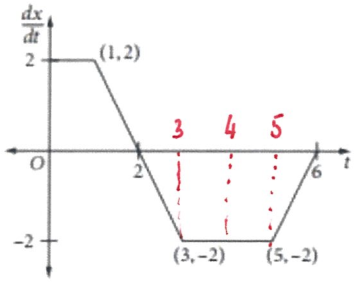
a) a is maximum at $t = 0$; it then decreases but is still positive until $t = 2$. So velocity is maximum at $t = 2$

b) at $t = 4$, as $v = \int_0^4 a \, dt = 0$

c) x is maximum when $v = 0$ and $a < 0$, which occurs at $t = 4$

OTHER EXAMPLES OF MOTIONS

10 The graph shows the velocity $\frac{dx}{dt}$ of a particle as a function of time. Initially the particle is at the origin.



(a) At what time is the displacement x from the origin a maximum?

(b) At what time does the particle return to the origin? Justify your answer.

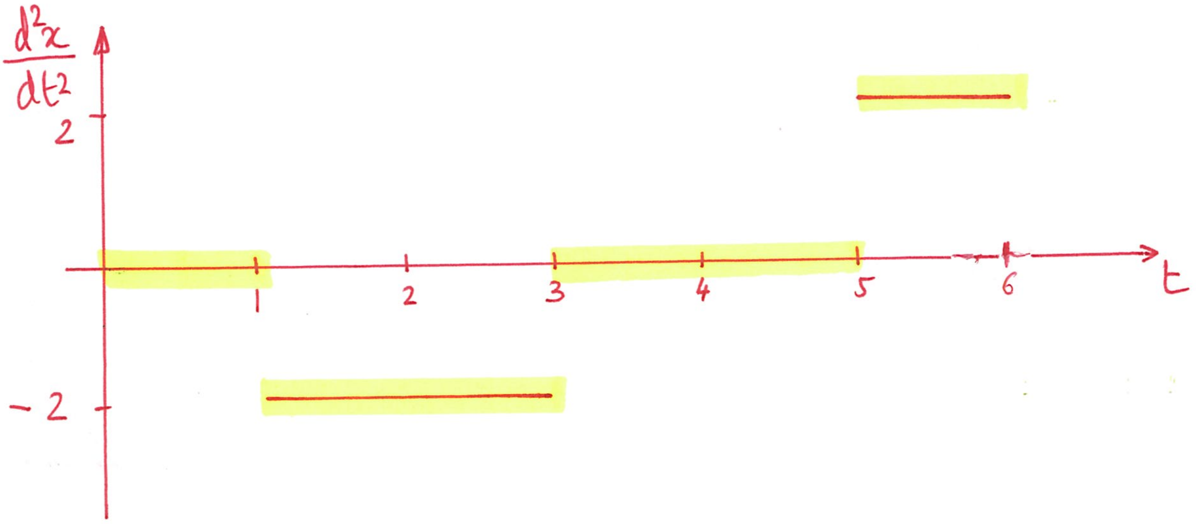
(c) Draw a sketch of the acceleration $\frac{d^2x}{dt^2}$ as a function of time for $0 \le t \le 6$.

a) $\frac{dx}{dt} > 0$ until $t = 2$, so until that time, the particle moves away from the origin. After $t = 2$, $\frac{dx}{dt} < 0$, so it comes back towards the origin.

So $t = 2$

b) $x(t) = \int_0^k v(t)\,dt = 0$ when $k = 4$, as the areas below the curve and $y = 0$ (for $t < 2$), and when $t > 2$, the area between $y = 0$ and the curve, are then equal. So $t = 4$

c)



OTHER EXAMPLES OF MOTIONS

11 The table shows the velocity (in metres per second) of a moving object, evaluated at one-second intervals.

t	0	1	2	3	4	5	6
v	0	4.6	5.7	8.0	9.9	12.7	18.2

Use the trapezoidal rule to estimate the distance travelled over the time interval $0 \le t \le 6$.

Trapezoidal rule (taken from the formula sheet)

$$\int_a^b f(x)\,dx \simeq \frac{b-a}{2n}\left\{f(a)+f(b)+2\left[f(x_1)+f(x_2)+\dots+f(x_{n-1})\right]\right\}$$

Here $b = 6 = x_6$, $a = 0 = x_0$ so $n = 6$

$$\int_0^6 f(x)\,dx \simeq \frac{6-0}{2\times 6}\left\{f(0)+f(6)+2\left[f(1)+f(2)+f(3)+f(4)+f(5)\right]\right\}$$

$$\int_0^6 f(x)\,dx \simeq \frac{1}{2}\left\{0+18.2+2\left[4.6+5.7+8.0+9.9+12.7\right]\right\}$$

$$\int_0^6 f(x)\,dx \simeq \frac{1}{2}\times 100 \quad \text{so} \quad 50$$

The distance travelled over the time interval $0 \leqslant t \leqslant 6$ is approximately 50 m

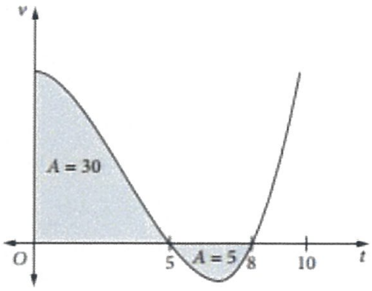
OTHER EXAMPLES OF MOTIONS

13 The velocity–time graph at right shows the first 10 seconds of motion of an object moving in a straight line. The graph also shows the areas between the curve and the t-axis from $t = 0$ to $t = 5$ and from $t = 5$ to $t = 8$.

(a) Find the distance travelled by the object before it first comes to rest.

(b) Find the total distance travelled by the object before it comes to rest for the second time.

(c) If the initial displacement is 20 metres, find the displacement when the object comes to rest for the second time.



a) $\dot{x}(t) = \frac{d\,x(t)}{dt}$ ∴ $x(t) = \int \dot{x}(t)\,dt = \int_0^5 \dot{x}(t)\,dt = 30$ m

as This is The area underneath the curve.

b) $x(t) = \int_0^8 \dot{x}(t)\,dt = \int_0^5 \dot{x}(t)\,dt + \int_5^8 \dot{x}(t)\,dt = 30 + 5 = 35$ m

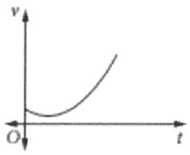
positive as we are asked about the "total distance" (≠ displacement)

c) Displacement $= 20 + 30 - 5 = 45$ m

as we need to consider negative values as we are asked about displacement.

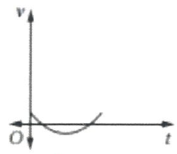
14 Each graph below shows the velocity–time relationship for an object moving in a straight line. In which case does the object change its direction only once?

A



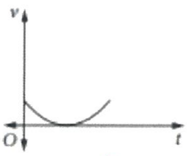
No change of direction

B



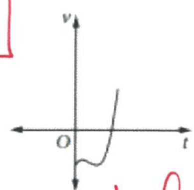
2 changes.

C



No change $\frac{dx}{dt}$ stays positive

$\boxed{\text{D}}$



1 change so $\boxed{\text{D}}$