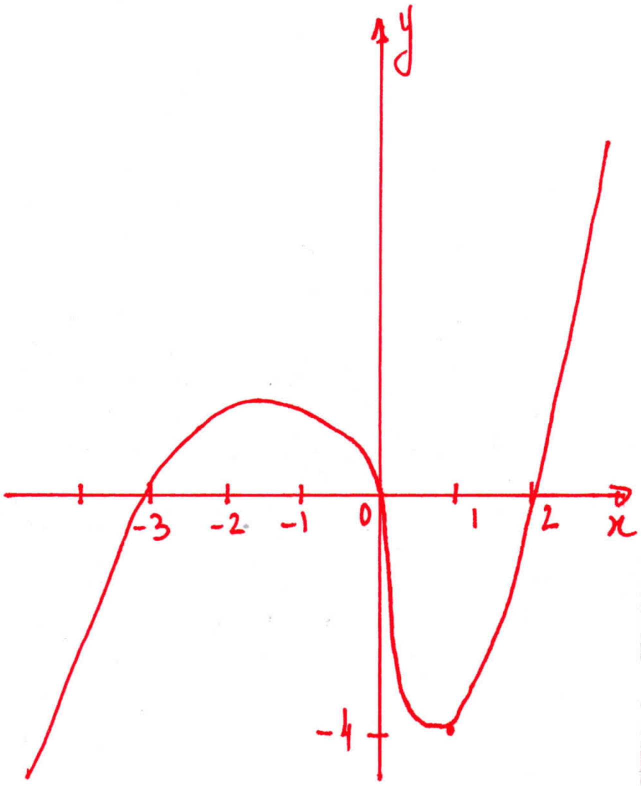
POLYNOMIAL FUNCTIONS

1 Sketch the graph of each function

$f(x) = x(x-2)(x+3)$

The graph cuts the x-axis at $x = 0$, $x = 2$ and $x = -3$



The coefficient of x^3 is > 0,

$\therefore \lim\limits_{x \to +\infty} f(x) = +\infty$

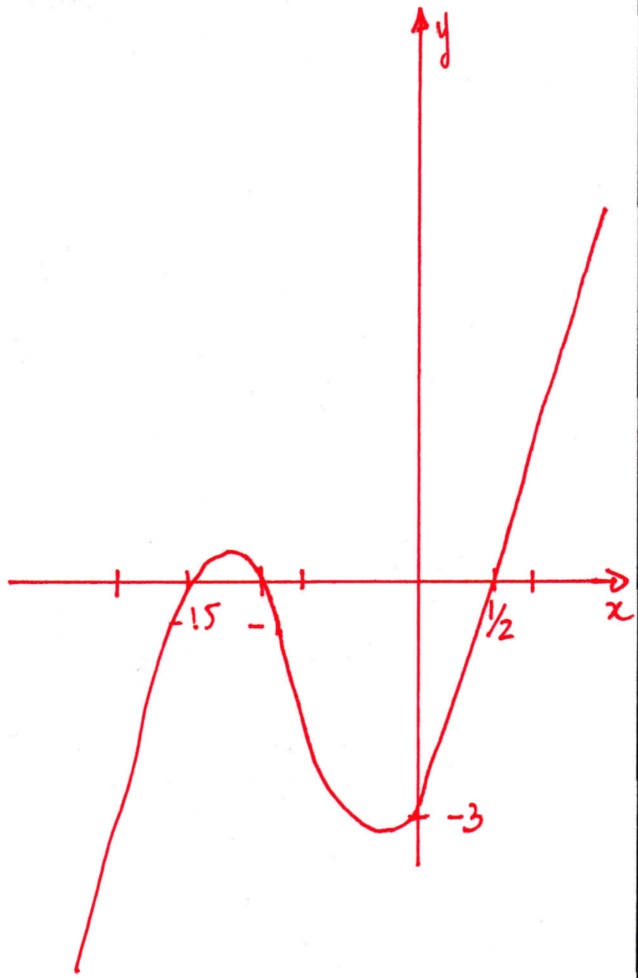
We can calculate a few points in between the roots

$f(1) = 1(1-2)(1+3) = -4$

$f(x) = 2(x - \frac{1}{2})(x+1)(2x+3)$

* The graph cuts the x-axis at $x = \frac{1}{2}$, $x = -1$ and $x = -1.5$
* the coefficient of x^3 is positive,

  $\therefore \lim\limits_{x \to +\infty} f(x) = +\infty$
* $f(0) = 2\left(-\frac{1}{2}\right) \times 1 \times 3 = -3$



POLYNOMIAL FUNCTIONS

4 **(a)** Find the linear factors of $x^3 - 5x^2 + 8x - 4$. $= P(x)$

(b) Find the values of x for which: **(i)** $x^3 - 5x^2 + 8x - 4 = 0$ **(ii)** $x^3 - 5x^2 + 8x - 4 > 0$

(c) Sketch the graph of f where $f(x) = x^3 - 5x^2 + 8x - 4$.

a) We try a few values at random

$P(1) = 1^3 - 5\times 1^2 + 8\times 1 - 4 = 1-5+8-4 = 0$ YES!

we found one root, so we can now factorise the cubic polynomial

$P(x) = (x-1)(x^2-4x+4)$

For the quadratic $\Delta = 16 - 4\times 4 = 0$

So two double roots $x = \dfrac{4}{2} = 2$

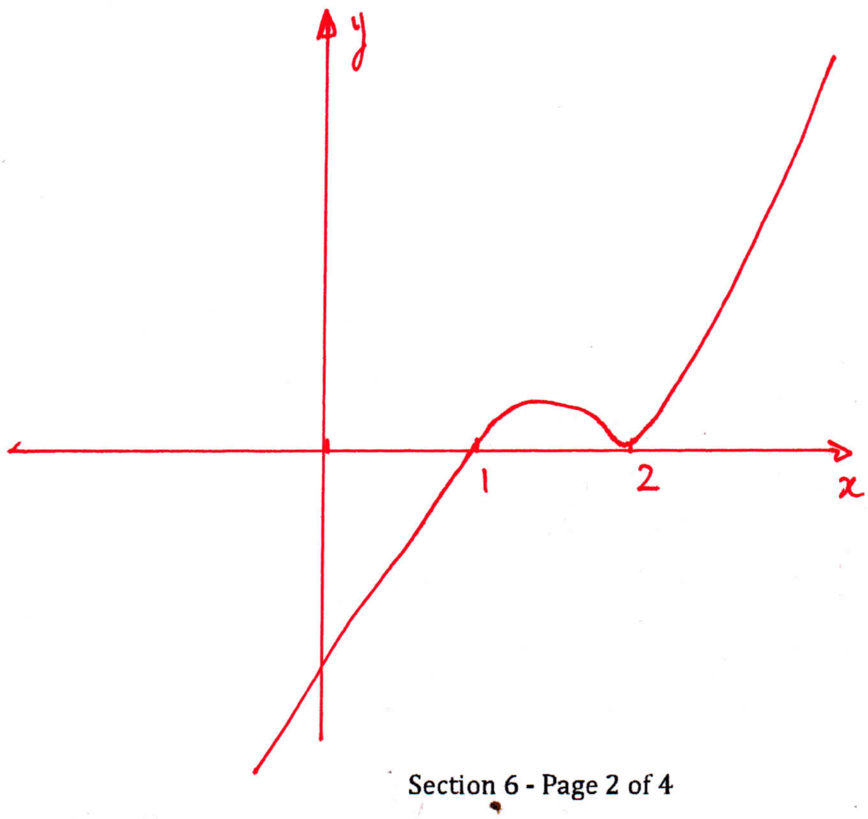
$P(x) = (x-1)(x-2)^2$

b) i) $x = 1$, $x = 2$ (double root)

ii) the coefficient of x^3 is positive, $\therefore \lim\limits_{x\to+\infty} P(x) = +\infty$

$P(x) > 0$ for $x > 1$ (As the root 2 has multiplicity 2, the graph doesn't cross the x-axis).

c)



POLYNOMIAL FUNCTIONS

8 Show that the graph of f, where $f(x) = x^3 - 8$, cuts the x-axis at one point only.

$f(x) = 0$ when $x^3 - 8 = 0 \iff x^3 = 8 \iff x = 2$

So the graph of f cuts the x-axis only for $x = 2$

9 Show that the graph of f, where $f(x) = x^3 - x^2 - 8x + 12$, cuts the x-axis at one point and touches it at another. Find the values of x at these points.

We look for the roots of the polynomial, trying a few values at random for the first one.

$f(1) = 1^3 - 1^2 - 8 \times 1 + 12 = 1 - 1 - 8 + 12 \neq 0$

$f(2) = 2^3 - 2^2 - 8 \times 2 + 12 = 8 - 4 - 16 + 12 = 0$ YES!

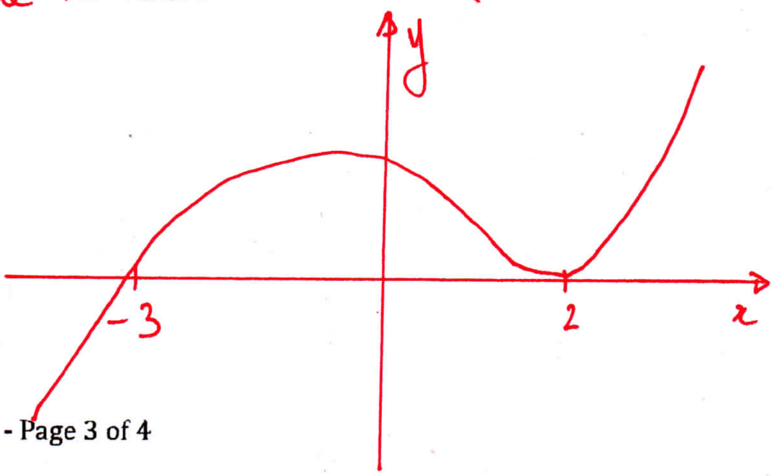
So the polynomial can be factorised as:

$f(x) = (x-2)(x^2 + x - 6)$

For the quadratic $\Delta = 1 - 4 \times (-6) = 25 = 5^2$

$x_1 = \dfrac{-1+5}{2} = 2 \qquad x_2 = \dfrac{-1-5}{2} = -3$

So the polynomial crosses the x-axis at $x = 2$ (double root) and $x = -3$.



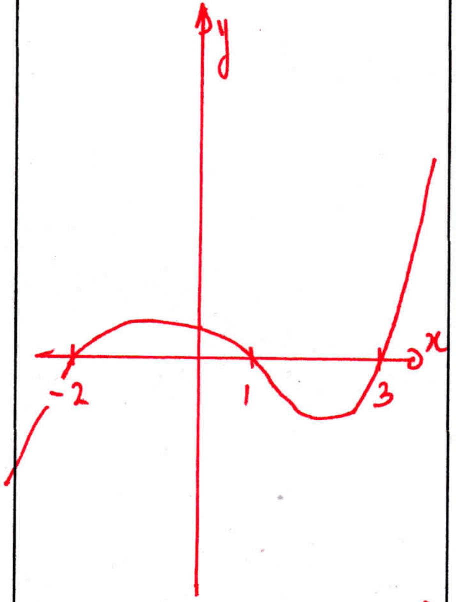
POLYNOMIAL FUNCTIONS

10 Sketch graphs of each function. For what values of x is each function positive?

(a) $y=(x-1)(x+2)(x-3)$

Roots are $1, -2, 3$

$-2, 1, 3$



$f(x)>0$ for

$x>3$ and

$-2<x<1$

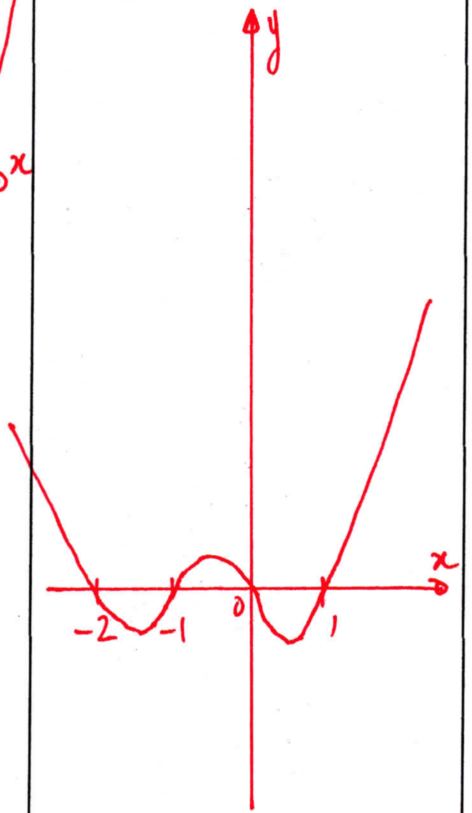
(c) $y=x(x^2-1)(x+2)$

Roots are $x=0$,

$x=-2$, $x=1$ and

$x=-1$

So $-2, -1, 0, 1$



$f(x)>0$ when

$x<-2, -1<x<0$,

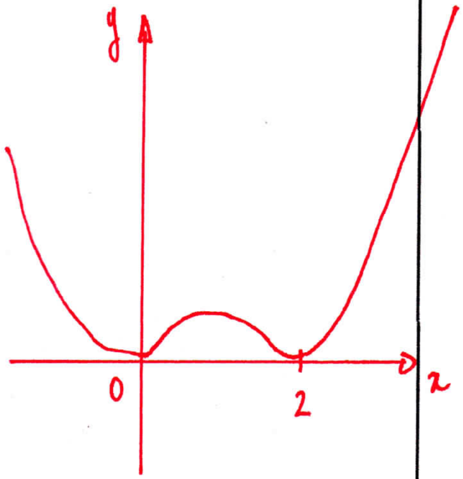
$x>1$

(d) $y=x^2(x-2)^2$

Roots are $x=0$,

$x=2$ (each of

multiplicity 2)



$f(x)>0$ for all $x \in \mathbb{R}$