3 Simplify these expressions using the index laws.

a $n^4 \times n^5 \times n^2$

$= n^9 \times n^2$

$= n^{11}$

b $p^8 \div p^3 \times p^5$

$= p^5 \times p^5$

$= p^{10}$

c $y^{13} \div y^4 \div y^2$

$= y^9 \div y^2$

$= y^7$

5 Simplify:

a $3a^4 \times 5a^3 \times 2a$

$= 30\,a^7 \times a$

$= 30\,a^8$

b $30g^{10} \div 6g^3 \times 4g^5$

$= 5g^7 \times 4g^5$

$= 20\,g^{12}$

c $100x^{13} \div 2x \div 5x^4$

$= 50\,x^{12} \div 5x^4$

$= 10\,x^8$

d $(3u^5)^3 \times 2u^4$

$= 27u^{15} \times 2u^4$

$= 54\,u^{19}$

e $56d^{19} \div (2d^4)^3$

$= 56\,d^{19} \div 8d^{12}$

$= 7\,d^7$

f $(4k^6)^5 \div (2k^2)^5$

$= 1024\,k^{30} \div 32\,k^{10}$

$= 32\,k^{20}$

g $\dfrac{9v^8 \times 8v^6}{12v^{11}}$

$= \dfrac{72\,v^{14}}{12\,v^{11}} = 6v^3$

h $\dfrac{8m^{11} \times 10m^2}{5m \times 4m^3}$

$= \dfrac{80\,m^{13}}{20\,m^4}$

$= 4\,m^9$

i $\dfrac{(12b^5)^2}{4b^2 \times (3b^3)^2}$

$= \dfrac{144\,b^{10}}{4b^2 \times 9b^6}$

$= \dfrac{144\,b^{10}}{36\,b^8} = 4b^2$

6 Simplify:

a $\dfrac{(m^6)^3 \times m^4}{m^7}$

$= \dfrac{m^{18} \times m^4}{m^7} = m^{15}$

b $\dfrac{(t^8)^5 \times (t^2)^3}{t^{10}} = \dfrac{t^{40} \times t^6}{t^{10}}$

$= \dfrac{t^{46}}{t^{10}} = t^{36}$

c $\dfrac{c^{30}}{(c^2)^4 \times c^6} = \dfrac{c^{30}}{c^8 \times c^6}$

$= c^{16}$

d $\dfrac{s^{15} \times (s^4)^4}{(s^4)^3 \times s^7} = \dfrac{s^{15} \times s^{16}}{s^{12} \times s^7}$

$= \dfrac{s^{31}}{s^{19}} = s^{12}$

e $(k^9)^5 \div \left(\dfrac{k^4}{k}\right)^7 = k^{45} \div (k^3)^7$

$= k^{45} \div k^{21} = k^{24}$

f $\left(\dfrac{h^{11}}{h^3}\right)^3 \times \dfrac{h^5}{h^3} = (h^8)^3 \times h^2$

$= h^{24} \times h^2 = h^{26}$

a $\dfrac{(3u^4)^2 \times 6u^{10}}{u^4 \times (3u^2)^3 \times u^2} = \dfrac{9u^8 \times 6u^{10}}{u^4 \times 27u^6 \times u^2}$

$= \dfrac{54\,u^{18}}{27\,u^{12}} = 2u^6$

b $\dfrac{10(pq)^2 \times 20p^{20}q^{14}}{(2p^6q^3)^2 \times 5p^5q}$

$= \dfrac{10\,p^2q^2 \times 20\,p^{20}\,q^{14}}{4\,p^{12}\,q^6 \times 5\,p^5\,q}$

$= \dfrac{200\,p^{22}\,q^{16}}{20\,p^{17}\,q^7} = 10p^5q^9$

8 Simplify each of the following expressions and evaluate where possible.

a $2^a \times 2^b$ $= 2^{a+b}$

b $k^m \times k^n$ $= k^{m+n}$

c $5^x \times 5$ $= 5^{x+1}$

d $3^n \times 3^n$ $= 3^{n+n} = 3^{2n}$

e $2^{5t} \times 2^t$ $= 2^{5t+t} = 2^{6t}$

f $p^q \times p^q \times p^q$ $= p^{3q}$

g $5^{m+1} \times 5^{m+3}$ $= 5^{m+1+m+3} = 5^{2m+4}$

h $a^{2b+5} \times a^{5-2b}$ $= a^{2b+5+5-2b} = a^{10}$

m $\frac{6^{e+3}}{6^3}$ $= 6^e$

n $\frac{5^{x+2}}{5^x}$ $= 5^2 = 25$

o $\frac{m^{p+q}}{m^{p-q}}$ $= m^{p+q-p+q} = m^{2q}$

p $\frac{x^{7y+6}}{x^{4+7y}}$ $= x^{7y+6-4-7y} = x^2$

q $(2^a)^b$ $= 2^{ab}$

r $(n^5)^k$ $= n^{5k}$

s $(e^f)^f$ $= e^{f^2}$

t $(p^{2q})^6$ $= p^{12q}$

u $(5^3)^{7u}$ $= 5^{21u}$

v $(6^{2a})^{3b}$ $= 6^{6ab}$

w $(2^x)^{y+z}$ $= 2^{x(y+z)}$

x $(a^{m-n})^k$ $= a^{(m-n)k}$

9 Half of $2^{2n}$ would be: $2^{2n} \div 2 = 2^{2n-1}$

A $2^n$ B $1^{2n}$ C $1^n$ D $2^{2n-1}$

10 Find one quarter of $2^n$ in index form.

$\frac{1}{4} \times 2^n = \frac{1}{2^2} \times 2^n = 2^{n-2}$

4 Evaluate:

a $6^0 + 4 = 1 + 4 = 5$

b $5 - 12^0 = 5 - 1 = 4$

c $4 \times 7^0 = 4$

d $5^0 + 3^0 = 1 + 1 = 2$

e $8a^0 - 2 = 8 - 2 = 6$

f $9p^0 \times 5 = 9 \times 5 = 45$

g $20 \div 4t^0 = 20 \div 4 = 5$

h $3 - 10x^0 = 3 - 10 = -7$

i $4k^0 + 7m^0 = 4 + 7 = 11$

j $p^0 + q^0 + r^0 = 3$

k $x^0 - y^0 - z^0 = 1 - 1 - 1 = -1$

l $3 + 9u^0 - 4^0 = 3 + 9 - 1 = 11$

m $6a^0 - b^0 + 3c^0 = 6 - 1 + 3 = 8$

n $14m^0 + (14m)^0 - 8n^0 = 14 + 1 - 8 = 7$

o $5^0 + (5a)^0 + 5a^0 = 1 + 1 + 5 = 7$

4 Write each expression with a positive index.

a $m^{-1} = \frac{1}{m}$ b $p^{-1} = \frac{1}{p}$ c $h^{-2} = \frac{1}{h^2}$ d $n^{-3} = \frac{1}{n^3}$ e $e^{-6} = \frac{1}{e^6}$ f $y^{-4} = \frac{1}{y^4}$

5 Write each of these with a negative index.

a $\frac{1}{a} = a^{-1}$ b $\frac{1}{x} = x^{-1}$ c $\frac{1}{c^2} = c^{-2}$ d $\frac{1}{u^3} = u^{-3}$ e $\frac{1}{p^5} = p^{-5}$ f $\frac{1}{y^8} = y^{-8}$

**11** Write each of these in surd form.

a $a^{\frac{3}{2}} = \sqrt{a^3}$ b $m^{\frac{2}{3}} = \sqrt[3]{m^2}$ c $p^{\frac{4}{3}} = \sqrt[3]{p^4}$ d $n^{\frac{3}{5}} = \sqrt[5]{n^3}$

e $k^{\frac{5}{6}} = \sqrt[6]{k^5}$ f $y^{\frac{7}{2}} = \sqrt{y^7}$ g $x^{-\frac{1}{2}} = \frac{1}{\sqrt{x}}$ h $c^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{c}}$

i $e^{-\frac{3}{2}} = 1/e^{3/2} = 1/\sqrt{e^3}$ j $s^{-\frac{2}{3}} = 1/s^{2/3} = 1/\sqrt[3]{s^2}$ k $v^{-\frac{4}{5}} = 1/v^{4/5} = 1/\sqrt[5]{v^4}$ l $b^{-\frac{7}{6}} = 1/b^{7/6} = 1/\sqrt[6]{b^7}$

**12** Write each of these in index form.

a $x\sqrt{x} = x^{3/2}$ b $x^2\sqrt{x} = x^{5/2}$ c $x \times \sqrt[3]{x} = x^{4/3}$ d $x^2 \times \sqrt[3]{x} = x^{7/3}$

e $x \times \sqrt[5]{x} = x^{6/5}$ f $x^2 \times \sqrt[4]{x} = x^{9/4}$ g $x^3\sqrt{x} = x^{3+\frac{1}{2}} = x^{7/2}$ h $x^4 \times \sqrt[7]{x} = x^{29/7}$

i $\frac{1}{\sqrt{x}} = x^{-1/2}$ j $\frac{1}{\sqrt[3]{x}} = x^{-1/3}$ k $\frac{1}{\sqrt[5]{x}} = x^{-1/5}$ l $\frac{1}{\sqrt[7]{x}} = x^{-1/7}$

m $\frac{1}{\sqrt[x]{x}} = x^{-1/x}$ n $\frac{1}{x \times \sqrt[3]{x}} = x^{-4/3}$ o $\frac{1}{x^2 \times \sqrt{x}} = x^{-5/2}$ p $\frac{1}{x^2 \times \sqrt[3]{x}} = x^{-7/3}$

**13** Write each of these in the form $a^m \times \sqrt[n]{a}$, where $m$ and $n$ are positive integers.

a $a^{\frac{3}{2}} = a \times a^{1/2} = a\sqrt{a}$ b $a^{\frac{5}{2}} = a^2 \times a^{1/2} = a^2\sqrt{a}$ c $a^{\frac{4}{3}} = a \times a^{1/3} = a\sqrt[3]{a}$ d $a^{\frac{5}{3}} = a \times a^{2/3} = a\sqrt[3]{a^2}$

**14** a Which is greater, $2^{\frac{1}{2}}$ or $3^{\frac{1}{3}}$? [HINT: Raise each number to the power of 6.]

$\therefore (2^{1/2})^6 = 2^{6/2} = 2^3 = 8$ whereas $(3^{1/3})^6 = 3^{6/3} = 3^2 = 9$

so $3^{1/3}$ is greater than $2^{1/2}$

Solve the following exponential equations

| | | |
|---|---|---|
| $2^x = 32 = 2^5$ so $x = 5$ | $2^x = 512 = 2^9$ so $x = 9$ | $3^x = 81 = 3^4$ so $x = 4$ |
| $7^x = 2401 = 7^4$ so $x = 4$ | $5^x = 625 = 5^4$ so $x = 4$ | $6^x = 216 = 6^3$ so $x = 3$ |
| $3^x = \frac{1}{9} = \frac{1}{3^2} = 3^{-2}$ so $x = -2$ | $2^x = \frac{1}{16} = \frac{1}{2^4} = 2^{-4}$ so $x = -4$ | $8^x = \frac{1}{64} = \frac{1}{8^2} = 8^{-2}$ so $x = -2$ |

| | | |
|---|---|---|
| $49^x = 7$ $= \sqrt{49} = 49^{1/2}$ so $x = 1/2$ | $128^x = 2 \Leftrightarrow (2^7)^x = 2$ $\Leftrightarrow 2^{7x} = 2$ so $7x = 1$ $x = 1/7$ | $81^x = \frac{1}{27} \Leftrightarrow (3^4)^x = \frac{1}{3^3} = 3^{-3}$ so $4x = -3$ $x = -3/4$ |
| $9^x = 27 \Leftrightarrow (3^2)^x = 3^3$ so $2x = 3$ $x = 3/2$ | $16^x = 8 \Leftrightarrow (2^4)^x = 2^3$ so $4x = 3$ $x = 3/4$ | $5^x = 5\sqrt{5} = 5^{1+1/2} = 5^{3/2}$ $x = 3/2$ |
| $4^x = \sqrt{2} \Leftrightarrow (2^2)^x = 2^{1/2}$ so $2x = \frac{1}{2}$ $x = \frac{1}{4}$ | $9^x = \frac{1}{\sqrt{3}} \Leftrightarrow (3^2)^x = \frac{1}{3^{1/2}} = 3^{-1/2}$ so $2x = -1/2$ $x = -1/4$ | $2^{x+1} = 16 = 2^4$ $x+1 = 4$ so $x = 3$ |
| $3^{x-1} = 243 = 3^5$ $x - 1 = 5$ $x = 6$ | $3^{2x-1} = 9 = 3^2$ $2x - 1 = 2$ $2x = 3$ $x = 3/2$ | $(\frac{1}{4})^x = 64 \Leftrightarrow 4^{-x} = 4^3$ $-x = 3$ $x = -3$ |

a) $8^{x-1} = 2^{x+2} \Leftrightarrow (2^3)^{x-1} = 2^{x+2}$

$3(x-1) = x+2$

$2x - 3 = 2$

$2x = 5$ $\boxed{x = 5/2}$

b) $144^{2x+1} = 12^{5x-1} \Leftrightarrow (12^2)^{2x+1} = 12^{5x-1}$

$2(2x+1) = 5x - 1$

$4x + 2 = 5x - 1$

$2 = x - 1$

so $\boxed{x = 3}$

c) $5^{3x+1} = 25^{x+1}$ $5^{3x+1} = (5^2)^{x+1}$

$3x + 1 = 2(x+1)$

$3x + 1 = 2x + 2$

$x + 1 = 2$

$\boxed{x = 1}$

d) $4^{-x+1} = 2^{2x} \Leftrightarrow (2^2)^{-x+1} = 2^{2x}$

$\Leftrightarrow 2^{-2x+2} = 2^{2x}$

$-2x + 2 = 2x$

$4x = 2$ $\boxed{x = 1/2}$

e) $64^a = 8^{a+2} \Leftrightarrow (8^2)^a = 8^{a+2}$

$2a = a + 2$

$\boxed{a = 2}$

f) $2^{m+1} = 16^{m+7} = (2^4)^{m+7} = 2^{4m+28}$

$m + 1 = 4m + 28$

$m = 4m + 27$

$-3m = 27$ $\boxed{m = -9}$

g) $36^{2x} = 216^{x-1} \Leftrightarrow (6^2)^{2x} = (6^3)^{x-1}$

$4x = 3(x-1) = 3x - 3$

$\boxed{x = -3}$

h) $3^{a-7} = 27^{2a} = (3^3)^{2a} = 3^{6a}$

$a - 7 = 6a$

$-7 = 5a$

So $\boxed{a = -7/5}$