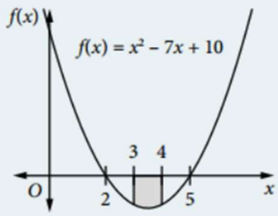
Example 13

Calculate the area of the region bounded by the graph of the parabola $f(x) = x^2 - 7x + 10$, the x-axis, and the ordinates $x = 3$ and $x = 4$.

Solution

Sketch the region:



$$\text{Area} = \left|\int_3^4 \left(x^2 - 7x + 10\right)dx\right|$$

$$= \left|\left[\frac{x^3}{3} - \frac{7x^2}{2} + 10x\right]_3^4\right|$$

$$= \left|\left(\frac{64}{3} - 56 + 40\right) - \left(9 - \frac{63}{2} + 30\right)\right|$$

$$= 2\frac{1}{6}$$

Odd and even functions

For an even function, the area bounded by the curve $y = f(x)$, the x-axis and the ordinates $x = a$ and $x = -a$ is:

$$\text{Area} = \left|\int_{-a}^{a} f(x)\,dx\right| = 2\left|\int_0^a f(x)\,dx\right|$$

For an odd function, the area bounded by the curve $y = f(x)$, the x-axis and the ordinates $x = a$ and $x = -a$ is:

$$\text{Area} = \left|\int_{-a}^{0} f(x)\,dx\right| + \left|\int_0^a f(x)\,dx\right| = 2\left|\int_0^a f(x)\,dx\right|$$

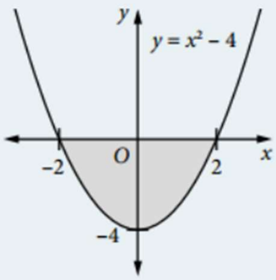
Example 14

Find the area of the region bounded by the curve $y = f(x)$, the x-axis and the ordinates $x = -2$ and $x = 2$, for:

(a) $f(x) = x^2 - 4$ **(b)** $f(x) = x^3$

Solution

(a) Sketch the region:



$f(x) = x^2 - 4$ is an even function.

$$\int_{-2}^{0}(x^2 - 4)\,dx = \int_{0}^{2}(x^2 - 4)\,dx$$

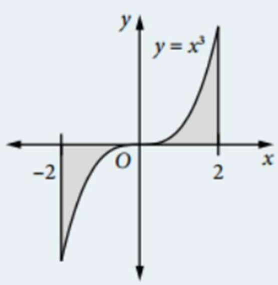
$$\therefore \text{Area} = 2\left|\int_{0}^{2}(x^2 - 4)\,dx\right|$$

$$= 2\left|\left[\frac{x^3}{3} - 4x\right]_0^2\right|$$

$$= 2\left|\frac{8}{3} - 8\right| = 2\left|-\frac{16}{3}\right|$$

$$= 10\frac{2}{3}\text{ units}^2$$

(b) Sketch the region:



$f(x) = x^3$ is an odd function.

$$\int_{-2}^{0} x^3\,dx = -\int_{0}^{2} x^3\,dx \quad \text{and} \quad \int_{-2}^{2} x^3\,dx = 0$$

$$\therefore \text{Area} = \left|\int_{-2}^{0} x^3\,dx\right| + \left|\int_{0}^{2} x^3\,dx\right| = 2\left|\int_{0}^{2} x^3\,dx\right|$$

$$= 2\left|\left[\frac{x^4}{4}\right]_0^2\right|$$

$$= 2|4 - 0|$$

$$= 8\text{ units}^2$$