ABSOLUTE VALUE FUNCTIONS

12 Solve for x:

(a) $|x-2|=3$

$x-2=\pm 3$

So $x=3+2=5$

OR $x=-3+2=-1$

(b) $|x+3|=7$

$x+3=\pm 7$

So $x=7-3=4$

OR $x=-7-3=-10$

(c) $|4-x|=5$

$4-x=\pm 5$

So $x=4-5=-1$

OR $x=4+5=9$

(d) $|x+7|=2$

$x+7=\pm 2$

So $x=2-7=-5$

OR $x=-2-7=-9$

(m) $|3x+1|=0$

$3x+1=0$

$x=-1/3$

(n) $|6x+1|=7$

$6x+1=\pm 7$

$6x=\pm 7-1$

$x=\frac{\pm 7-1}{6}$

So $x=\frac{7-1}{6}=1$

OR $x=\frac{-7-1}{6}=-\frac{4}{3}$

(o) $|4x-1|=0$

$4x-1=0$

$x=1/4$

(p) $|2x-9|=13$

$2x-9=\pm 13$

$x=\frac{\pm 13+9}{2}$

So $x=\frac{13+9}{2}=11$

OR $x=\frac{-13+9}{2}=-2$

14 Solve:

(a) $|x-1|<3$

So $-3<x-1<3$

$-3+1<x<3+1$

$-2<x<4$

(b) $|y+2|>4$

So $y+2>4$ OR $y+2<-4$

$y>2$ OR $y<-6$

(c) $|t-6|\leq 2$

$-2\leq t-6\leq 2$

$-2+6\leq t\leq 2+6$

$4\leq t\leq 8$

(d) $|x+4|\geq 2$

So $x+4\geq 2$ OR $x+4\leq -2$

$x\geq -2$ OR $x\leq -6$

27 $|x-1|<-2$

$|x-1|$ is positive whereas -2 is negative. A positive number cannot be less than a negative one. So there's no solutions

28 $|2x-3|\leq 5$

$-5\leq 2x-3\leq 5$

$-2\leq 2x\leq 8$

$-1\leq x\leq 4$

29 $|3x+2|<2$

$-2<3x+2<2$

$-4<3x<0$

$-4/3<x<0$

30 $|x^2-1|\leq 4$

$-4\leq x^2-1\leq 4$

$-3\leq x^2\leq 5$

But x^2 positive

So $x^2\leq 5$

$-\sqrt{5}\leq x\leq \sqrt{5}$

ABSOLUTE VALUE FUNCTIONS

40 For the following values of x and y, verify that **(i)** $|xy| = |x| \times |y|$ and **(ii)** $|x+y| \le |x| + |y|$.

(a) $x = 5, y = 2$ **(b)** $x = 3, y = -2$ **(c)** $x = -6, y = 8$ **(d)** $x = -4, y = -3$

a) i) $|5 \times 2| = 10 = |5| \times |2|$ indeed

ii) $|5+2| = 7$ whereas $|x| + |y| = |5| + |2| = 7$ so true

b) i) $|3 \times (-2)| = |-6| = 6$ whereas $|x| \times |y| = |3| \times |-2| = 3 \times 2 = 6$ so true

ii) $|3 + (-2)| = |1| = 1$ whereas $|x| + |y| = |3| + |-2| = 3 + 2 = 5$ so true

c) i) $|x \times y| = |-6| \times |8| = 6 \times 8 = 48$ whereas $|x| \times |y| = |-6| \times |8| = 6 \times 8 = 48$ so true

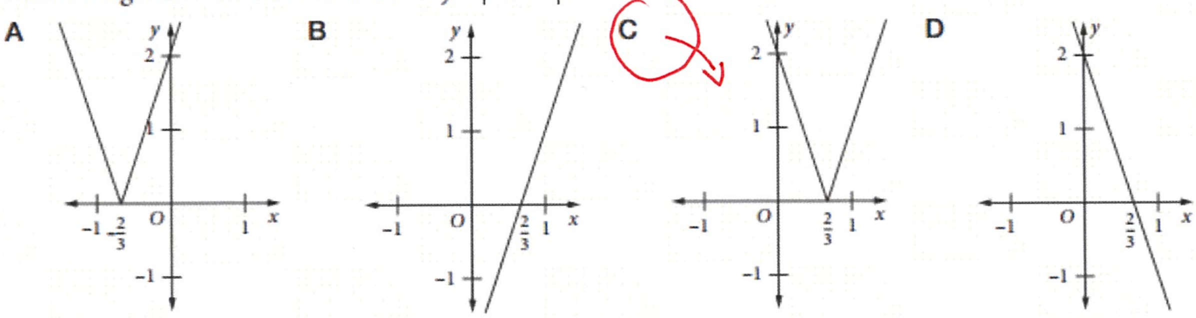
ii) $|x + y| = |-6 + 8| = |2| = 2$ whereas $|x| + |y| = |-6| + |8| = 6 + 8 = 14$ so true

d) i) $|x \times y| = |(-4) \times (-3)| = |12| = 12$ whereas $|x| \times |y| = |-4| \times |-3| = 4 \times 3 = 12$ so true

ii) $|x + y| = |-4 - 3| = |-7| = 7$ whereas

$|x| + |y| = |-4| + |-3| = 4 + 3 = 7$ so true

2 Which diagram is the correct sketch of $y = |3x - 2|$?



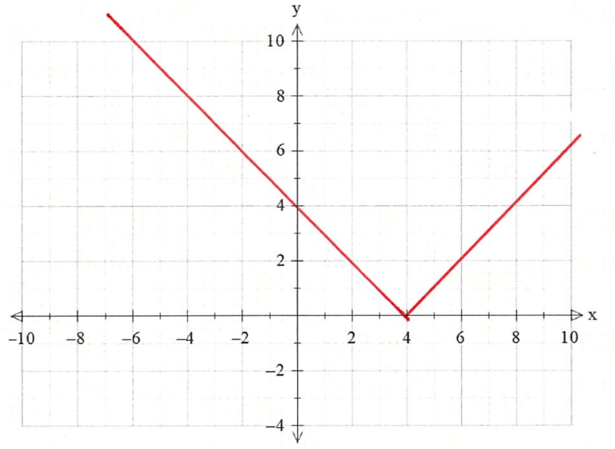
$$|3x - 2| = \begin{cases} 3x - 2 & \text{if } 3x - 2 \geq 0 \text{ (i.e. } x \geq 2/3) \\ 2 - 3x & \text{if } 3x - 2 \leq 0 \text{ (i.e. } x \leq 2/3) \end{cases}$$

ABSOLUTE VALUE FUNCTIONS

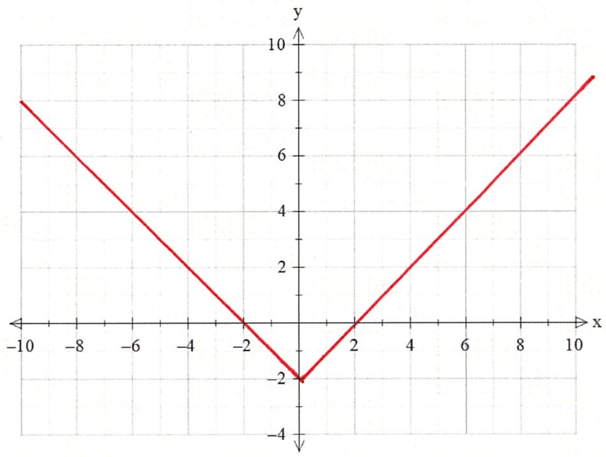
1 Sketch the graphs of the following absolute value functions defined for all x and state the range in each case.

(a) $f(x)=|x-4|$

$f(x)=\begin{cases} x-4 & \text{if } x-4>0 \text{ (i.e. } x>4) \\ 4-x & \text{if } x-4<0 \text{ (i.e. } x<4) \end{cases}$

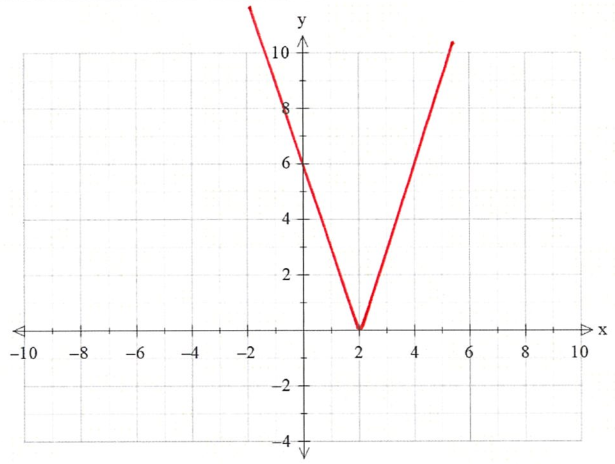


(b) $g(x)=|x|-2$



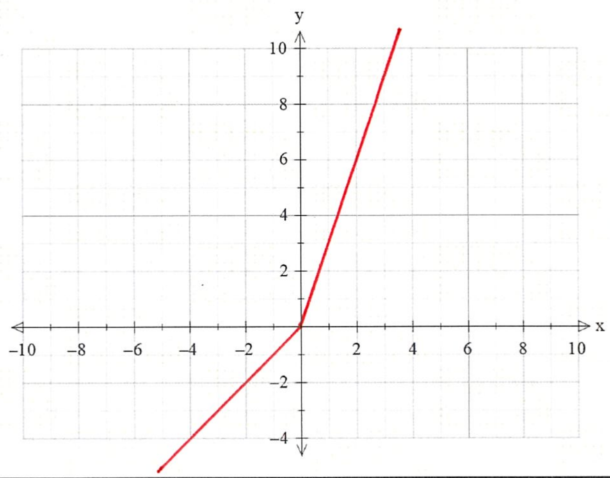
(e) $h(x)=|3x-6|$

$h(x)=\begin{cases} 3x-6 & \text{if } 3x-6\geq 0 \text{ (i.e. } x\geq 2) \\ 6-3x & \text{if } 3x-6\leq 0 \text{ (i.e. } x\leq 2) \end{cases}$



(h) $f(x)=2x+|x|$

$f(x)=\begin{cases} 2x+x=3x & \text{if } x>0 \\ 2x-x=x & \text{if } x<0 \end{cases}$



3 State the largest possible domain for:

(a) $f(x)=\sqrt{x-2}+\sqrt{3-x}$

Domain for $\sqrt{x-2}$ is $x\geq 2$

Domain for $\sqrt{3-x}$ is $x\leq 3$

So domain for $f(x)$ is $2\leq x\leq 3$

(b) $f(x)=\frac{x}{|x|}$

$|x|$ must be different of 0 as it's at the denominator

So Domain is $\mathbb{R}-\{0\}$

(or also noted $(-\infty,0)\cup(0,+\infty)$)

ABSOLUTE VALUE FUNCTIONS

4 State whether the following functions are odd, even or neither, defined on their largest possible domain.

(a) $f(x) = x$ **(b)** $f(x) = x + 1$ **(c)** $f(x) = |x|$

a) $f(-x) = -x = -f(x)$
so odd

Domain is $\mathbb{R}$

$f(-x) = -x+1$
neither odd or even

Domain is $\mathbb{R}$

$f(-x) = |-x| = |x| = f(x)$
so f is even

Domain is $\mathbb{R}$

(g) $f(x) = \sqrt{4 - x^2}$ **(h)** $f(x) = \dfrac{x}{x^2 - 1}$ **(i)** $f(x) = x^2 + x$

$f(-x) = \sqrt{4 - (-x)^2}$
$f(-x) = \sqrt{4 - x^2} = f(x)$
so even

Domain is $[-2, 2]$

$f(-x) = \dfrac{-x}{(-x)^2 - 1}$
$f(-x) = \dfrac{-x}{x^2 - 1} = -f(x)$
so odd

Domain is $\mathbb{R} - \{-1, 1\}$

$f(-x) = (-x)^2 + (-x) = x^2 - x$
so neither odd or even

Domain is $\mathbb{R}$

5 Find the largest possible range for the following functions:

(a) $f(x) = (x - 3)^2$ **(b)** $f(x) = x + |x|$ **(c)** $f(x) = \sqrt{16 - x^2}$ **(d)** $f(x) = 16 - x^2$

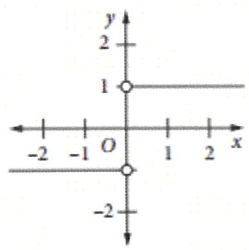
Range is $\mathbb{R}^+$

when $x > 0$ $f(x) = 2x$
when $x < 0$ $f(x) = 0$
So Range is $\mathbb{R}^+$

Domain is $[-4, 4]$
So Range is $[0, 4]$

Range is $(-\infty, 16]$

9 For the given graph, state whether each statement is correct or incorrect.



(a) The domain is real x, $x \neq 0$. TRUE

(b) The range is real y, $-1 < y < 1$. FALSE, it's $y = 1$ or $y = -1$

(c) The gradient of the function is zero.

(d) The equation of the function could be $y = \dfrac{|x|}{x}$.

c) TRUE

d) if $x > 0$, $f(x) = \dfrac{|x|}{x} = \dfrac{x}{x} = 1$

if $x < 0$ $f(x) = \dfrac{|x|}{x} = \dfrac{-x}{x} = -1$ so d) is TRUE