HARDER EXPONENTIAL GROWTH AND DECAY

1 N is decreasing according to the equation $\frac{dN}{dt} = -0.4(N - 30)$. If $N = 60$ when $t = 0$:

(a) show that $N = 30 + Ae^{-0.4t}$ is a solution of this equation, where A is a constant

(b) calculate the value of N when $t = 5$.

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3 The original temperature of a body is 120°C, the temperature of its surroundings is 50°C and the body cools to 70°C in 10 minutes. Assuming Newton's law of cooling, i.e. $\frac{dT}{dt} = -k(T - 50)$ where T is the temperature of the body at time t, find:

(a) the temperature after 20 minutes **(b)** the time taken to cool to 60°C.

4 If $N = 70$ when $t = 0$, which expression is the correct solution to $\frac{dN}{dt} = -0.5(N - 20)$?

A $N = 20 + 50e^{0.5t}$ **B** $N = 20 + 50e^{-0.5t}$ **C** $N = 20 - 50e^{0.5t}$ **D** $N = 20 - 50e^{-0.5t}$

5 A metal bar has a temperature of 1230°C and cools to 1030°C in 10 minutes when the surrounding temperature is 30°C. Assume Newton's law of cooling, i.e. $\frac{dT}{dt} = -k(T - 30)$ where T is the temperature of the body at time t.

(a) Show that $T = 30 + 1200e^{-kt}$ satisfies both Newton's law of cooling and the initial conditions.

(b) Find the temperature after 20 minutes.

(c) Find the time taken to cool from 1230°C to 80°C.

7 A body whose temperature is 180°C is immersed in a liquid that is at 60°C. In 1 minute the temperature of the body has fallen to 120°C. Assume Newton's law of cooling, i.e. $\frac{dT}{dt} = -k(T - 60)$ where T is the temperature of the body at time t.

(a) Show that $T = 60 + 120e^{-kt}$ satisfies both Newton's law of cooling and the initial conditions.

(b) At what time would the temperature of the body have fallen to 90°C?

8 A current of i amperes (or 'amps') flows through a coil of inductance L henrys and resistance R ohms. The current at any time is given by $i=\frac{E}{R}\left(1-e^{\frac{-Rt}{L}}\right)$, where E is the electromotive force (i.e. the voltage) in volts. Show that $L\frac{di}{dt}+Ri=E$.

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9 A vessel is filled at a variable rate so that the volume of liquid in the vessel at any time t is given by $V = A(1 - e^{-kt})$.

(a) Show that $\frac{dV}{dt} = k(A - V)$.

(b) If a quarter of the vessel is filled in the first 5 minutes, what fraction is filled in the next 5 minutes?

(c) Show that $\lim_{t \to \infty} V = A$.

10 A rectangular vessel is divided into two equal compartments by a vertical porous membrane. Liquid in one compartment, initially at a depth of 20 cm, flows into the other compartment, initially empty, at a rate proportional to the difference between the levels in each compartment. The differential equation for this process is $\frac{dx}{dt} = k(20 - 2x)$, where x cm is the depth of the liquid in one of the vessels at any time t minutes.

(a) Show that $x = 10(1 - e^{-2kt})$.

(b) If the level in the second compartment rises 2 cm in the first 5 minutes, at what time will the difference in levels be 2 cm?

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11 In a certain chemical process, the amount y grams of a certain substance at time t hours is given by the formula $y = 3 + e^{-kt}$.

(a) Show that $\frac{dy}{dt} = -k(y - 3)$.

(b) If initially y decreases at a rate of 0.08 grams per hour, find the value of k.

(c) Find the rate of change when $y = 3.5$.

(d) What values can y take?