COMPLEX NUMBERS AND POLYNOMIAL EQUATIONS

1 Use the factor theorem to show that:

(a) $z - i$ is a factor of $z^3 + 2iz^2 + 3i$ **(b)** $z - 3$ is a factor of $z^2 - (5 - i)z + 6 - 3i$

(c) $z + 2 - i$ is a factor of $2z^3 + 3z^2 - (5 + 2i)z - 17 - 9i$

(d) $z - 3 + \sqrt{2}i$ is a factor of $2z^4 - 12z^3 + 23z^2 - 6z + 11$.

2 Given $P(z) = z^3 - z^2 - z + a$, what is the value of a if $P\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = 0$?

A 2 **B** 1 **C** -2 **D** -1

3 Given $P(z) = z^4 - 2z^3 + az - 9$, find the value of a if $P\left(1 + \sqrt{2}i\right) = 0$.

6 Factorise each polynomial over the set of complex numbers:

(a) $z^2 + 2z + 3$ **(b)** $2z^2 - 2z + 1$ **(c)** $2z^3 - 3z^2 + 2z - 3$

8 Factorise $z^4 - 16$ over:

(a) the set of integers **(b)** the set of complex numbers.

10 Factorise $z^3 - 4z^2 + 9z - 10$ over:

(a) the set of real numbers **(b)** the set of complex numbers.

12 When factorised over the set of complex numbers, $z^4 + 2z^2 + 1$ becomes:

A $(z^2 - 1)^2$ **B** $(z - i)^2(z + 1)^2$ **C** $(z^2 + 1)^2$ **D** $(z - i)^2(z + i)^2$

13 Factorise $z^6 - 1$ over:

(a) the set of real numbers **(b)** the set of complex numbers.

14 Factorise $z^5 + 3z^4 - z - 3$ over:

(a) the set of real numbers **(b)** the set of complex numbers.