SKETCHING BASIC FUNCTIONS

You should be able to sketch simple linear functions from either the gradient-intercept form or the general form of the equation. You also need to be able to quickly and neatly sketch the power functions, such as $f(x)=x^2, f(x)=-x^2, f(x)=x^3, f(x)=-x^3, f(x)=x^4, f(x)=-x^4$, $f(x)=\frac{1}{x}$ and $f(x)=\frac{1}{x^2}$. You should already be familiar with many of these.

The following examples should also be graphed using graphing software. You may need to rewrite each equation in the form $y=f(x)$.

Example 7

Sketch each straight line. State the gradient and both axis intercepts of each.

(a) $y=2x+1$ **(b)** $2x+3y-6=0$ **(c)** $y=4-x$

Solution

(a) $y=2x+1$

Find the value of y for three different values of x: $(0,1), (2,5), (-1,-1)$

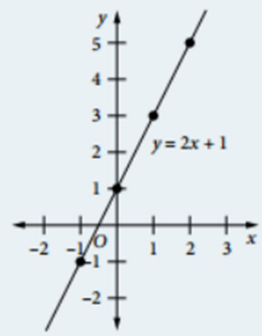
Plot these points on the number plane. Join them to obtain the line.

OR

From the form of the equation, recognise that the y-intercept is 1 and the gradient is 2.

Because the line passes through $(0,1)$ it also passes through $(1, 1+2=3)$ and $(2, 3+2=5)$. This is because the gradient is 2, which means that as x increases by 1, y increases by 2. Plot and join the points.

Gradient = 2, x-intercept = –0.5, y-intercept = 1.



(b) $2x+3y-6=0$

Find the value of y for three different values of x: $(0,2), (3,0), (-3,4)$

Plot these points on the number plane. Join them to obtain the line.

OR

Rewrite the equation in the gradient-intercept form: $y=\frac{-2x}{3}+2$

The gradient is a fraction, so this is not so convenient.

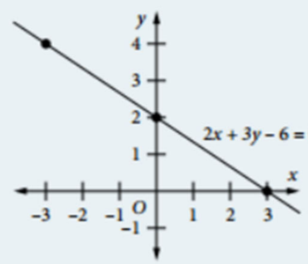
OR

Rewrite the equation by putting the constant term on the RHS of the equation and dividing by 6: the equation becomes $\frac{x}{3}+\frac{y}{2}=1$.

This shows that the x-intercept is 3 and the y-intercept is 2. Draw a line through these intercept points to obtain the graph.

Because the line falls as x increases, the gradient is negative.

$$\text{Gradient} = -\frac{y\text{-intercept}}{x\text{-intercept}} = -\frac{2}{3}, \ x\text{-intercept} = 2, \ y\text{-intercept} = 3.$$



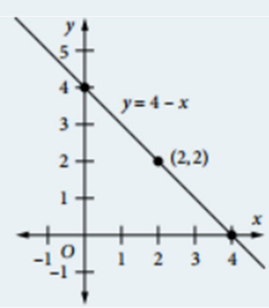
(c) $y=4-x$

From the equation:

gradient = –1, x-intercept = 4, y-intercept = 4.

Use this information to sketch the graph.

If you use this method, you should also find the coordinates of a third point to check that you haven't made a mistake, e.g. $(2,2)$.



Example 8

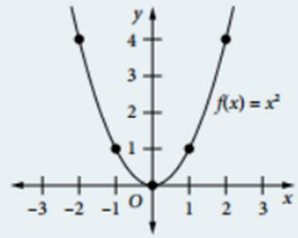
Sketch each function, showing any intercepts on the coordinate axes.

(a) $f(x) = x^2$ **(b)** $f(x) = -x^3$ **(c)** $f(x) = \frac{1}{x}$

Solution

(a) $f(x) = x^2$ is a type of curve called a parabola. Create a table of values and plot the points.

x	−2	−1	0	1	2
$f(x)$	4	1	0	1	4



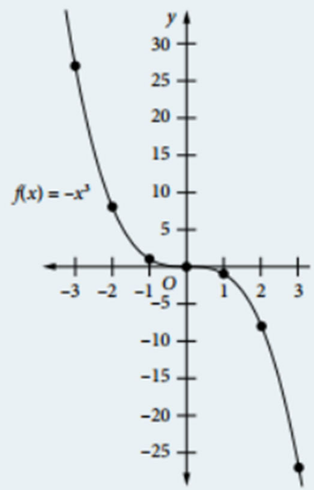
Note that $f(-x) = f(x)$. The curve is symmetrical about the y-axis. The curve passes through the point $(0, 0)$.

When $x > 0$, $f(x)$ increases as x increases, so we say that $f(x)$ is an increasing function for $x > 0$.

When $x < 0$, $f(x)$ decreases as x increases, so we say that $f(x)$ is a decreasing function for $x < 0$.

(b) $f(x) = -x^3$ is a type of curve called a cubic. Create a table of values and plot the points.

x	−3	−2	−1	0	1	2	3
$f(x)$	27	8	1	0	−1	−8	−27



Note that $f(-x) = -f(x)$. The curve has rotational or point symmetry about the origin.

The curve passes through the point $(0, 0)$.

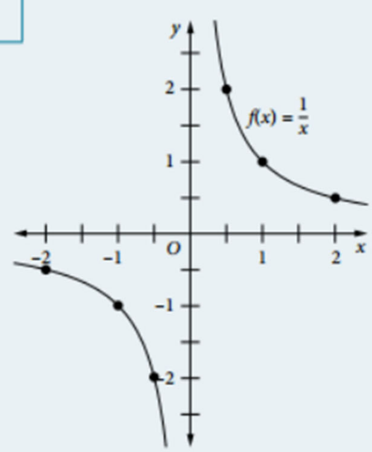
As x increases over the domain, the value of $f(x)$ decreases, so $f(x)$ is a decreasing function over its domain.

(c) $f(x) = \frac{1}{x}$ is a type of curve called a hyperbola. Create a table of values and plot the points.

x	−2	−1	−0.5	0	0.5	1	2
$f(x)$	−0.5	−1	−2	undefined	2	1	0.5

Note that $f(-x) = -f(x)$. The curve has rotational or point symmetry about the origin. Also note that $f(0)$ is undefined because $\frac{1}{0}$ does not exist.

The curve does not cut either axis.
As $x \to \pm\infty$, $f(x) \to 0$ and as $f(x) \to \pm\infty$, $x \to 0$.
The line $f(x) = 0$ is called a horizontal asymptote.
The line $x = 0$ is called a vertical asymptote.



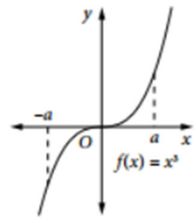
When $x < 0$, $f(x)$ decreases as x increases, so we say that $f(x)$ is a decreasing function for $x < 0$.

When $x > 0$, $f(x)$ decreases as x increases, so we say that $f(x)$ is a decreasing function for $x > 0$.

Thus $f(x)$ is a decreasing function over each part of its domain.

Odd and even functions

An **odd function** has the property that $f(-x) = -f(x)$. For example:

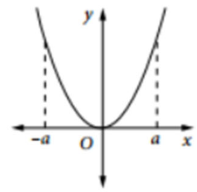


$$\begin{aligned} \text{If} \quad & f(x) = x^3 \\ \text{then} \quad & f(-x) = (-x)^3 \\ & = -x^3 \\ & = -f(x) \end{aligned}$$

Hence $f(x) = x^3$ is an odd function.

Because $f(x)$ and $f(-x)$ are opposite in sign, the graph of f for $x \le 0$ can be obtained by rotating the graph for $x \ge 0$ through an angle of 180° about the origin.

An **even function** has the property that $f(-x) = f(x)$. For example:



f(x) = x²

$$\begin{aligned} \text{If} \quad & f(x) = x^2 \\ \text{then} \quad & f(-x) = (-x)^2 \\ & = x^2 \\ & = f(x) \end{aligned}$$

Hence $f(x) = x^2$ is an even function.

The graph of an even function is symmetrical about the y-axis. The graph for $x \le 0$ can be obtained by reflecting the graph for $x \ge 0$ in the y-axis.

Note that the statement $f(-a) = f(a)$ implies that the function is defined at both $x = a$ and $x = -a$. The function $f(x) = x^2, x > 0$ is **not** an even function, because $f(-a)$ is not defined.

The properties of odd and even functions are useful when sketching the curves for these functions. After drawing a curve for $x \ge 0$, the other half of the curve can be drawn immediately from the odd or even symmetrical properties. Disappointingly, however, most functions are neither even nor odd.