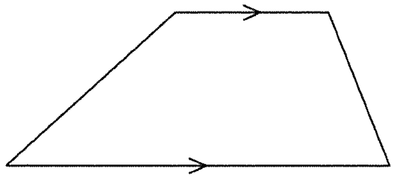
VECTORS IN GEOMETRIC PROOFS

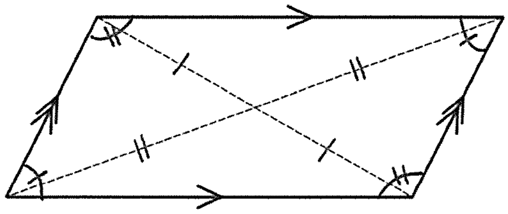
Many geometry theorems can be solved using vector methods.

Properties of triangles and special quadrilaterals:

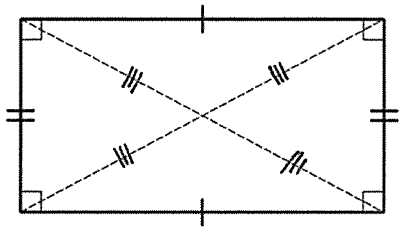
- a **trapezium** is a quadrilateral with one pair of opposite sides parallel



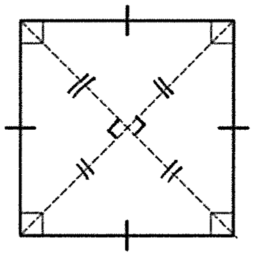
- a **parallelogram** is a quadrilateral with both pairs of opposite sides parallel. It has the properties that both pairs of opposite sides are equal, both pairs of opposite angles are equal and the diagonals bisect each other.



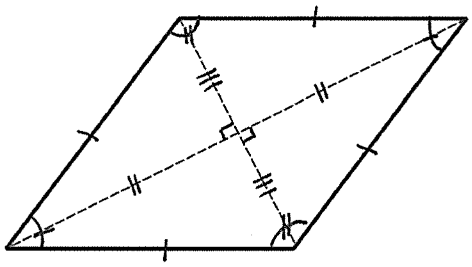
- a **rectangle** is a parallelogram where one angle is a right angle. It has all the properties of the parallelogram, as well as that each angle is 90°, the diagonals are equal, and each diagonal divides the rectangle into a pair of congruent triangles.



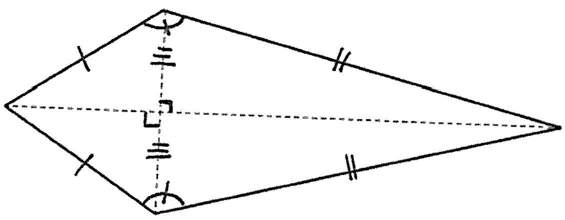
- a **square** is a rectangle with a pair of equal adjacent sides. It has all the properties of a rectangle, as well as that sides are of equal length, the diagonals bisect the angles of the square, the diagonals bisect each other at right angles, and the diagonals divide the square into four congruent right-angled triangles.



- a **rhombus** is a parallelogram with a pair of adjacent sides equal. It has all the properties of the parallelogram, as well as that all sides are of equal length, the diagonals bisect the angles of the rhombus, the diagonals bisect each other at right angles, and the diagonals divide the rhombus into four congruent right-angled triangles.



- a **kite** is a quadrilateral with two pairs of adjacent sides equal. The opposite angles between the pairs of non-equal sides are equal.



VECTORS IN GEOMETRIC PROOFS

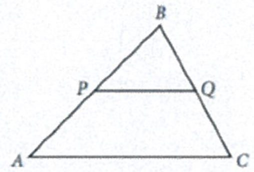
Vector proof involving sides of a triangle

Example 23

Use vector methods to prove that the line segment joining the midpoints of two sides of a triangle is parallel to the third side and half its length.

Solution

Consider ΔABC, where P and Q are the midpoints of sides $\overline{AB}$ and $\overline{BC}$ respectively.



Let $\overrightarrow{AB} = \underset{\sim}{a}$ and $\overrightarrow{BC} = \underset{\sim}{b}$.

P is the midpoint of $\overline{AB}$ and Q is the midpoint of $\overline{BC}$,

so $\overrightarrow{PB} = \frac{1}{2}\underset{\sim}{a}$ and $\overrightarrow{BQ} = \frac{1}{2}\underset{\sim}{b}$.

Express vectors required in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$: $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

$$= \underset{\sim}{a} + \underset{\sim}{b}$$

$$\overrightarrow{PQ} = \overrightarrow{PB} + \overrightarrow{BQ}$$

$$= \frac{1}{2}(\underset{\sim}{a} + \underset{\sim}{b})$$

$$\therefore \overrightarrow{PQ} = \frac{1}{2}\overrightarrow{AC}$$

Thus, $\overline{PQ}$ is parallel to $\overline{AC}$ and half the length of $\overline{AC}$.