FURTHER WORK WITH VECTORS - CHAPTER REVIEW

1 For each of the points, P, whose coordinates are given, find:

(i) an $\underset{\sim}{i}, \underset{\sim}{j}, \underset{\sim}{k}$ representation for the position vector $\overrightarrow{OP}$ **(ii)** the magnitude of $\overrightarrow{OP}$

(iii) a unit vector in the direction of $\overrightarrow{OP}$.

(a) $P(-1, 4, 2)$ **(b)** $P(3, 6, 8)$ **(c)** $P(-2, 2, -1)$

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2 Given $A(3, 3, 1)$, $B(-2, 1, -1)$, $C(1, 1, 1)$ and $D(2, 1, -2)$, find:

(a) the angle between $\overrightarrow{AB}$ and $\overrightarrow{CD}$

(b) the angle between $\overrightarrow{AC}$ and $\overrightarrow{BD}$

(c) the angle between $\overrightarrow{AD}$ and $\overrightarrow{BC}$.

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3 Determine whether the given sets of points are collinear.

(a) $A(1, 3, 2), B(3, 1, 4), C(5, -2, -6)$

(b) $D(1, 3, -4), E(3, -2, 2), F(3, 1, 5)$

4 Given $\underset{\sim}{a} = 2\underset{\sim}{i} + 3\underset{\sim}{j} - 4\underset{\sim}{k}$, $\underset{\sim}{b} = 3\underset{\sim}{i} - 5\underset{\sim}{j} - 4\underset{\sim}{k}$, $\underset{\sim}{c} = 2\underset{\sim}{i} + 6\underset{\sim}{j} + 3\underset{\sim}{k}$, find unit vectors $\hat{\underset{\sim}{a}}, \hat{\underset{\sim}{b}}, \hat{\underset{\sim}{c}}$.

5 If $\underset{\sim}{a} = 2\underset{\sim}{i} + 3\underset{\sim}{j} + 4\underset{\sim}{k}$, $\underset{\sim}{b} = 4\underset{\sim}{i} - \underset{\sim}{j} - 2\underset{\sim}{k}$ and $\underset{\sim}{c} = -5\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$, simplify:

(a) $(\underset{\sim}{a} \bullet \underset{\sim}{b})\underset{\sim}{c} + (\underset{\sim}{a} \bullet \underset{\sim}{c})\underset{\sim}{b}$ **(b)** $(\underset{\sim}{c} - \underset{\sim}{a}) \bullet \underset{\sim}{b}$ **(c)** $(\underset{\sim}{a} - \underset{\sim}{b}) \bullet (\underset{\sim}{b} - \underset{\sim}{c})$

6 The position vectors of the points P, Q and R are $8\underset{\sim}{i} - 4\underset{\sim}{j} - 3\underset{\sim}{k}$, $6\underset{\sim}{i} + 3\underset{\sim}{j} - 4\underset{\sim}{k}$ and $7\underset{\sim}{i} + 5\underset{\sim}{j} - 5\underset{\sim}{k}$ respectively. Find the angle between $\overrightarrow{PQ}$ and $\overrightarrow{QR}$.

7 Find a vector perpendicular to both $\underset{\sim}{u} = 4\underset{\sim}{i} - 7\underset{\sim}{j} + 4\underset{\sim}{k}$ and $\underset{\sim}{v} = -7\underset{\sim}{i} + 4\underset{\sim}{j} + 4\underset{\sim}{k}$.

8 Show that each given equation is the equation of a sphere and find the coordinates of its centre and the radius.

(a) $x^2 + y^2 + z^2 + 14x - 12y + 2z + 5 = 0$ **(b)** $x^2 + y^2 + z^2 - 6x + 2z + 6 = 0$

9 For the curves whose parametric equations are given, find:

(i) the Cartesian equation (ii) the vector equation.

(a) $x = 2t, y = t^2, t \in R$

(b) $x = \sec \theta, y = \tan \theta, -\frac{\pi}{2} < \theta < \frac{\pi}{2}$.

12 Find the vector equation of the line through $A(4, 3, 6)$ and $B(2, 5, 3)$.

13 Show that the line through the points $(1, -1, 1)$ and $(5, 3, 3)$ is perpendicular to the line through the points $(1, 1, 2)$ and $(4, -4, 6)$.

15 If $\underset{\sim}{a} = \underset{\sim}{i} + 2\underset{\sim}{j} - 3\underset{\sim}{k}$, $\underset{\sim}{b} = 5\underset{\sim}{i} + 2\underset{\sim}{j} - 4\underset{\sim}{k}$, $\underset{\sim}{c} = 2\underset{\sim}{i} - \underset{\sim}{j} - 4\underset{\sim}{k}$, find the values of p and q such that $\underset{\sim}{a} + p\underset{\sim}{b} + q\underset{\sim}{c}$ is parallel to the y-axis.

16 **(a)** Show that the points $O(0, 0, 0)$, $A(1, 1, 0)$, $B(1, 0, 1)$ and $C(0, 1, 1)$ are the vertices of a regular tetrahedron by finding the lengths of each of the six edges.

(b) Use the dot product to find the angle between any two edges.

(c) If M is the midpoint of BC, find the size of $\angle AMB$.

17 Relative to a fixed origin, the points A, B and C are defined respectively by the position vectors $\underset{\sim}{a} = \underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$, $\underset{\sim}{b} = 2\underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{c} = m\underset{\sim}{i}$, where m is a real constant.

(a) If $\angle ABC = \frac{\pi}{3}$, find m.

(b) If $\angle ABC = \frac{\pi}{2}$, find m.