# RESISTED MOTION

2 A particle has an initial velocity $U$. After travelling a distance $d$ in time $T$ along a straight horizontal path, its velocity is $V$. The retardation of the particle at any time is proportional to its mass and velocity at that time. Show that:

a) $V = U - kd$

b) $U = Ve^{kT}$

c) $U = Ve^{\frac{T(U-V)}{d}}$

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4 An object of mass $m$ falls from rest under constant gravitational force and against air resistance equal to $kv$, where $v$ is the speed and $k$ is a positive constant.

(a) Find its velocity at any time $t$.

(b) Sketch the velocity–time graph.

(c) Find the terminal velocity. Find the time taken to reach a speed $v_1$ where $v_1$ is one-quarter of the terminal velocity.

(d) Find the distance travelled when the speed $v_1$ is reached.

**4** A particle falls from rest under constant gravity and a resistance force, which is proportional to $m$ and to the square of the velocity. Find:

a) the equation of motion

b) the terminal velocity

c) the distance fallen as a function of the velocity

d) the distance fallen when half the terminal velocity is reached

e) the time taken to reach half the terminal velocity

9 A parachutist jumps from a stationary balloon at a great height. The parachute opens after 10 seconds. Assume the air resistance produces a retardation proportional to the mass of the parachutist and to the velocity, with a constant of proportionality $k = 0.1$ for the first 10 seconds (i.e. during freefall) and $k = 2$ after the parachute opens, find:

a) the parachutist's velocity after 10 seconds

b) the parachutist's velocity after 15 seconds

c) the parachutist's terminal velocity, i.e. the approximate velocity while floating to the ground.

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**10** A particle is projected vertically upwards against air resistance. Its acceleration at any time $t$ seconds after projection is given by $\ddot{x} = -\left(g + \frac{1}{10}v^2\right)$, where $v$ m s$^{-1}$ is the velocity. If the initial velocity is 20 m s$^{-1}$, find:

**(a)** the greatest height reached

**(b)** the time taken to reach the greatest height.

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**11** A particle is projected vertically upwards with initial speed $u$. Its acceleration is given by the differential equation $\ddot{x} = -(g + kv)$ where $v$ is the speed at any time $t$, $k$ is a positive constant and $kv$ is the retardation due to air resistance.

**(a)** Find the maximum height reached by the particle.

**(b)** Find the time taken to reach the maximum height.

**(c)** Write the differential equation for the downward motion.

**(d)** Show that the particle returns to its point of projection with a speed $V$ given by:

$$k(u + V) = g\log_e\left[\frac{g + ku}{g - kV}\right]$$

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**14** A particle of unit mass moves in a horizontal straight line against a resistance numerically equal to $v + v^3$, where $v$ is its velocity. Initially the particle is at the origin and is travelling with velocity $Q$, where $Q > 0$.

**(a)** Show that: $\tan^{-1} Q - \tan^{-1} v = \tan^{-1}\left[\dfrac{Q-v}{1+Qv}\right]$

**(b)** Show that $x = \tan^{-1}\left[\dfrac{Q-v}{1+Qv}\right]$, where $x$ is the displacement.

**(c)** Show that $t = \dfrac{1}{2}\log_e\left[\dfrac{Q^2(1+v^2)}{v^2(1+Q^2)}\right]$, where $t$ is the elapsed time when the particle is travelling with velocity $v$.

**(d)** Find $v^2$ as a function of $t$.

**(e)** Find the limiting values of $v$ and $x$ as $t \to \infty$.

**15** A particle of unit mass is projected vertically upwards in a medium in which the retardation due to resistance is $0.1v$. It is allowed to fall back to its point of projection. The initial speed of projection is $V_0$ and the final speed on return is $V_F$. Show that:

**(a)** the equation of motion on the upwards journey is $\ddot{x} = -(g + 0.1v)$

**(b)** the maximum height reached is $h = 10V_0 + 100g \log_e\left(\frac{10g}{10g + V_0}\right)$

**(c)** the time taken to reach the highest point is $T_1 = 10 \log_e\left(\frac{10g + V_0}{10g}\right)$

**(d)** the equation of motion on the downwards journey is $\ddot{x} = g - 0.1v$

**(e)** the time taken on the downwards journey is $T_2 = 10 \log_e\left(\frac{10g}{10g - V_F}\right)$

**(f)** by analysis of the downwards journey, $h = -10V_F + 100g \log_e\left(\frac{10g}{10g - V_F}\right)$

**(g)** the total time of the motion is $T = \frac{V_0 + V_F}{g}$.