1 Express each of the following as a simpler trigonometric function.

(a) $\sin(\pi - x)$ **(b)** $\cos\left(\frac{\pi}{2} - x\right)$ **(c)** $\tan(2\pi - x)$

(d) $\cos(\pi + x)$ **(e)** $\sin(2\pi - x)$ **(f)** $\cot\left(\frac{\pi}{2} - x\right)$

a) $\sin(\pi - x) = \sin x$

b) $\cos\left(\frac{\pi}{2} - x\right) = \sin x$

c) $\tan(2\pi - x) = \tan(-x) = \frac{\sin(-x)}{\cos(-x)} = \frac{-\sin x}{\cos x} = -\tan x$

d) $\cos(\pi + x) = -\cos x$

e) $\sin(2\pi - x) = \sin(-x) = -\sin x$

f) $\cot\left(\frac{\pi}{2} - x\right) = \frac{\cos\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right)} = \frac{\sin x}{\cos x} = \tan x$

2 For any angle θ, $\cos(\pi - \theta)$ is equal to:

A $-\cos\theta$ **B** $\cos\theta$ **C** $\sin\theta$ **D** $-\sin\theta$

$\cos(\pi - \theta) = -\cos\theta$

3 Indicate whether each statement is correct or incorrect.

(a) $\cos\left(\frac{\pi}{2} - \theta\right) = \cos\theta$ **(b)** $\cos(2\pi - \theta) = \cos\theta$ **(c)** $\sin(\pi + \theta) = \sin\theta$ **(d)** $\sin(2\pi - \theta) = -\sin\theta$

incorrect correct incorrect correct

4 If $\sin x = 0.2$, write the value of:

(a) $\sin(\pi - x)$ **(b)** $\sin(2\pi - x)$ **(c)** $\sin(-x)$ **(d)** $\cos\left(\frac{\pi}{2} - x\right)$ **(e)** $\sin(\pi + x)$ **(f)** $\operatorname{cosec} x$

a) $\sin(\pi - x) = \sin x = 0.2$

b) $\sin(2\pi - x) = \sin(-x) = -\sin x = -0.2$

c) $\sin(-x) = -\sin x = -0.2$

d) $\cos\left(\frac{\pi}{2} - x\right) = \sin x = 0.2$

e) $\sin(\pi + x) = -\sin x = -0.2$

f) $\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{0.2} = \frac{1}{\frac{1}{5}} = 5$

6 If $\cos x = c$, express the following in terms of c:

(a) $\sec x$ **(b)** $\cos(-x)$ **(c)** $\cos(\pi - x)$ **(d)** $\cos(2\pi - x)$ **(e)** $\sec(-x)$ **(f)** $\cos(\pi + x)$

a) $\sec x = \frac{1}{\cos x} = \frac{1}{c}$

b) $\cos(-x) = \cos x = c$

c) $\cos(\pi - x) = -\cos x = -c$

d) $\cos(2\pi - x) = \cos(-x) = \cos x$

e) $\sec(-x) = \frac{1}{\cos(-x)} = \frac{1}{\cos x} = \frac{1}{c}$

f) $\cos(\pi + x) = -\cos x = -c$

7 Write the exact value of:

(a) $\sin\frac{\pi}{2}$ (b) $\cos\frac{2\pi}{3}$ (c) $\tan\frac{5\pi}{6}$ (d) $\cos\pi$

(e) $\sec\frac{3\pi}{4}$ (f) $\cot\frac{5\pi}{6}$ (g) $\operatorname{cosec}\frac{\pi}{2}$ (h) $\sin\frac{2\pi}{3}$

a) $\sin \pi/2 = 1$ b) $\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$

c) $\tan\left(\frac{5\pi}{6}\right) = \frac{\sin(5\pi/6)}{\cos(5\pi/6)} = \frac{1/2}{-\sqrt{3}/2} = -\frac{1}{\sqrt{3}}$ d) $\cos\pi = -1$

e) $\sec\left(\frac{3\pi}{4}\right) = \frac{1}{\cos(3\pi/4)} = \frac{1}{(-\sqrt{2}/2)} = -\frac{2}{\sqrt{2}} = -\sqrt{2}$

f) $\cot(5\pi/6) = \frac{\cos(5\pi/6)}{\sin(5\pi/6)} = \frac{-\sqrt{3}/2}{1/2} = -\sqrt{3}$ g) $\operatorname{cosec}\frac{\pi}{2} = \frac{1}{\sin\pi/2} = 1$

h) $\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$

9 Write the exact value of:

(a) $\sin\frac{3\pi}{2}$ (b) $\tan\frac{5\pi}{3}$ (c) $\operatorname{cosec}\frac{11\pi}{6}$ (d) $\tan\frac{7\pi}{4}$

(e) $\cot\frac{7\pi}{4}$ (f) $\cos\frac{11\pi}{6}$ (g) $\sin\frac{5\pi}{3}$ (h) $\operatorname{cosec}\frac{5\pi}{3}$

a) $\sin(3\pi/2) = -1$ b) $\tan\left(\frac{5\pi}{3}\right) = \frac{\sin(5\pi/3)}{\cos(5\pi/3)} = \frac{-\sqrt{3}/2}{1/2} = -\sqrt{3}$

c) $\operatorname{cosec}\left(\frac{11\pi}{6}\right) = \frac{1}{\sin(11\pi/6)} = \frac{1}{(-1/2)} = -2$

d) $\tan\frac{7\pi}{4} = \frac{\sin(7\pi/4)}{\cos(7\pi/4)} = \frac{-\sqrt{2}/2}{\sqrt{2}/2} = -1$ e) $\cot\left(\frac{7\pi}{4}\right) = \frac{\cos 7\pi/4}{\sin 7\pi/4} = -1$

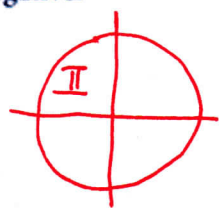
f) $\cos\left(\frac{11\pi}{6}\right) = \frac{\sqrt{3}}{2}$ g) $\sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

h) $\operatorname{cosec}\left(\frac{5\pi}{3}\right) = \frac{1}{\sin(5\pi/3)} = \frac{1}{-\frac{\sqrt{3}}{2}} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$

12 If θ is an angle in the 2nd quadrant, state whether the following are positive or negative:

(a) $\cos(\pi-\theta)$ **(b)** $\tan(\pi+\theta)$ **(c)** $\sin\left(\frac{\pi}{2}-\theta\right)$

(d) $\sin(2\pi-\theta)$ **(e)** $\cos(\pi+\theta)$ **(f)** $\tan\left(\frac{\pi}{2}-\theta\right)$



a) in I quadrant, $\therefore \cos(\pi-\theta) > 0$

b) $(\pi+\theta)$ in IV quadrant, $\therefore \tan(\pi+\theta) = \frac{\sin(\pi+\theta)}{\cos(\pi+\theta)}$ ($\sin(\pi+\theta) < 0$, $\cos(\pi+\theta) > 0$) < 0

c) $\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta \not< 0$

d) $\sin(2\pi-\theta) = \sin(-\theta) = -\sin\theta < 0$ as $\sin\theta > 0$

e) $\cos(\pi+\theta) = -\cos\theta \not> 0$ as $\cos\theta < 0$

f) $\tan\left(\frac{\pi}{2}-\theta\right) = \frac{\sin(\pi/2-\theta)}{\cos(\pi/2-\theta)} = \frac{\cos(\theta)}{\sin\theta}$ ($\cos\theta < 0$, $\sin\theta > 0$) $\not> 0$ < 0.

14 Using a diagram, find equivalent expressions for:

(a) $\cos\left(\frac{3\pi}{2}+x\right)$ **(b)** $\tan\left(\frac{3\pi}{2}-x\right)$ **(c)** $\sin\left(\frac{3\pi}{2}-x\right)$

a) $\cos\left(\frac{3\pi}{2}+x\right) = -\sin x$

b) $\tan\left(\frac{3\pi}{2}-x\right) = \tan\left(\pi+\frac{\pi}{2}-x\right) = \tan\left(\frac{\pi}{2}-x\right)$ as tan periodic & period π

$= \frac{\sin(\pi/2-x)}{\cos(\pi/2-x)} = \frac{\cos x}{\sin x} = \cot x$

c) $\sin\left(\frac{3\pi}{2}-x\right) = \sin\left(\pi+\frac{\pi}{2}-x\right) = -\sin\left(\frac{\pi}{2}-x\right) = -\cos x$

13 Solve, for $0 < x < 2\pi$:

(a) $\sin x = -\frac{\sqrt{3}}{2}$ (b) $\tan x = -1$ (c) $\cos x = -1$ (d) $\cot x = \sqrt{3}$

(e) $\sec x = -\sqrt{2}$ (f) $\sin x = \cos x$

a) $\sin x = -\frac{\sqrt{3}}{2}$ means either $x = \frac{4\pi}{3}$ or $x = \frac{5\pi}{3}$

b) $\tan x = -1$ means either $x = \frac{3\pi}{4}$ or $x = \frac{7\pi}{4}$

c) $\cos x = -1$ so $x = \pi$

d) $\text{cotan } x = \sqrt{3} = \frac{\cos x}{\sin x} = \frac{(\sqrt{3}/2)}{(1/2)} = \frac{(-\sqrt{3}/2)}{(-1/2)}$

so $x = \pi/6$ or $x = 7\pi/6$

e) $\sec x = -\sqrt{2} = \frac{1}{\cos x}$ so $\cos x = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

$x = \frac{3\pi}{4}$ or $x = \frac{5\pi}{4}$

f) $\sin x = \cos x \iff \tan x = 1$

$x = \pi/4$ or $x = 5\pi/4$

13 Solve, for $0 < x < 2\pi$:

(g) $\sin x = 0$ **(h)** $2\cos x + 1 = 0$

(i) $2\sin x = \sqrt{3}$ **(j)** $\sin x + \sqrt{3}\cos x = 0$ **(k)** $\operatorname{cosec} x = \sec x$

g) $\sin x = 0$ means $x = \pi$

h) $2\cos x + 1 = 0 \iff \cos x = -1/2$

so $x = 2\pi/3$ or $x = 4\pi/3$

i) $2\sin x = \sqrt{3} \iff \sin x = \frac{\sqrt{3}}{2}$

so $x = \frac{\pi}{3}$ or $x = \frac{2\pi}{3}$

j) $\sin x + \sqrt{3}\cos x = 0 \iff \sin x = -\sqrt{3}\cos x$

$\iff \tan x = -\sqrt{3} = \frac{-\sqrt{3}/2}{1/2} = \frac{\sqrt{3}/2}{-1/2}$

so $x = \frac{5\pi}{3}$ or $x = \frac{2\pi}{3}$

k) $\operatorname{cosec} x = \sec x \iff \frac{1}{\sin x} = \frac{1}{\cos x} \iff \sin x = \cos x$

$\iff \tan x = 1$

so $x = \pi/4$ or $x = 5\pi/4$