2 A bag contains four discs, each of a different colour. How many different arrangements can be made of four discs if:

(a) they are selected at random one at a time and placed in a row until the bag is empty

(b) one disc is selected at random, its colour is noted and it is put back in the bag before the next disc is selected?

a) 4 choices for the 1st, then 3 choices for the 2nd, then 2 choices for the 3rd, so $4! = 24$

b) $4^4 = 256$

3 How many different arrangements can be made of the five vowels *a, e, i, o, u*?

$5! = 120$

4 How many different four-digit numbers can be formed from the digits 2, 3, 4, 5, 6, 7, 8 if:

(a) none of the digits are repeated

(b) the digits may be repeated?

a) ${}^7P_4 = 840$

b) $7^4 = 2401$

5 How many four-digit or five-digit numbers greater than 4000 can be formed using 0, 2, 4, 5, 7 if:

(a) none of the digits are repeated
(b) the digits may be repeated
(c) none of the digits are repeated and the numbers formed are even?

a) 4 digits : can start by 4,5,7 (3 choices) then 4 choices, then 3 choices, then 2 choices so $3 \times 4! = 72$

5 digits: can start by 2,4,5,7 (4 choices) then 4,3,2,1 so $4 \times 4! = 96$

Total 168

b) 4 digits : 3 choices at first, then 5^3 so $3 \times 5^3 = 375$

5 digits : can start by 2,4,5,7 (4 choices) then 5^4, so $4 \times 5^4 = 2500$

so $\boxed{2875}$

c) 4 digits = 48 possibilities (8×6)

5 digits $= 10 \times 3! = 60$. Total 108

7 In how many ways can a cycling team of four riders be selected from a squad of ten riders?

$^{10}C_4 = 210$

9 A mixed volleyball team of eight players is selected from eight males and nine females. In how many ways can this be done if the team must have an equal number of male and female players?

So 4 male and 4 female.

$^8C_4 \times {}^9C_4 = 70 \times 126 = 8820$

10 A committee of five is to be selected from a class of 30 students.

(a) In how many ways can this be done?
(b) After the committee is selected, one person is elected chairperson and another is elected secretary. In how many ways can these positions be filled?

a) $^{30}C_5 = 142506$

b) $5 \times 4 = 20$

11 In how many ways can five people be seated in a five-seater car if only two people have a licence to legally occupy the driver's seat?

$2 \times 4! = 48$

12 **(a)** In how many ways can a committee of three parents and four teachers be chosen from six parents and eight teachers?

(b) The members of the committee are seated in a row of seven chairs on a stage. In how many different ways can they be seated?

(c) Find the number of ways of seating this committee if the parents refuse to sit at the ends of the row.

(d) What is the probability that for a chosen committee, the parents do not sit at the ends of the row?

a) ${}^6C_3 \times {}^8C_4 = 1{,}400$

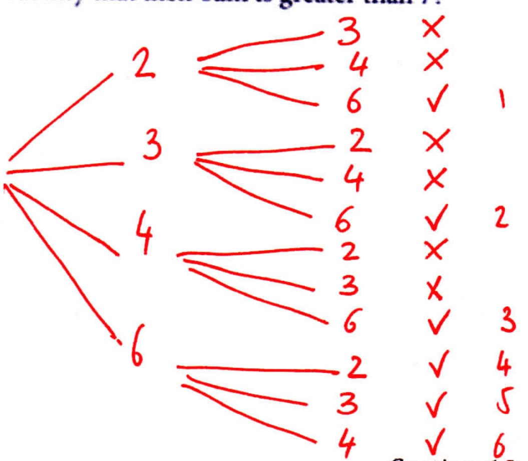
b) $7! = 5{,}040$

c) $4 \times 3 \times 5! = 1440$

d) P(parents do not sit at the ends) $= 1440/5040$

$= 2/7$

13 A box contains four marbles labelled 2, 3, 4, 6. If two marbles are drawn at random from the box, what is the probability that their sum is greater than 7?



$\frac{6}{12} = \frac{1}{2}$

14 The letters of the word CONSIDER are rearranged. What is the probability that the rearranged set of letters will begin with 'SI'?

$\frac{1}{8} \times \frac{1}{7} = \frac{1}{56}$

15 Three-digit numbers are formed using the digits 1, 2, 3, 4, 5, 6, 7. If none of the digits are repeated, what is the probability of forming an even number? E E E

$\frac{3}{7}$ as they are 3 possible digits out of 7

16 The letters of the word ARRANGEMENT are arranged in a row. 11 letters

(a) In how many ways can this be done using all the letters?

(b) What is the probability that the vowels are all together?

a) 2 As, 2 Rs, 2 Ns, 2 Es so $\frac{11!}{2!2!2!2!} = 2,494,800$

b) Vowels are 2 As and 2 Es.

How many ways for the vowels to be together?

$N = \frac{4!}{2!\,2!} \times \frac{8!}{2!\,2!} = 6 \times 10,080 = 60,480$

So $P = \frac{60,480}{2,494,800} = \frac{4}{165}$

17 Three cards are selected from a standard pack of 52 playing cards. What is the probability that at least one of the cards will be a King?

P(at least one card is a King) = 1 − P(no card is a King)

$$= 1 - \frac{^{48}C_3}{^{52}C_3}$$

$$= 1 - \frac{4324}{5525} = \frac{1201}{5525} \approx 0.2174$$

19 Seven-digit numbers are formed using all the digits 2, 2, 3, 4, 6, 7, 7.

(a) How many different numbers can be formed?

(b) What is the probability that the number formed is odd?

(c) What is the probability that the number formed is greater than 4 000 000?

(d) What is the probability that the number formed is less than 7 000 000?

a) 7 numbers, but numbers 2 and 7 are repeated, so $\frac{7!}{2!2!} = 1260$

b) There are 3 odd numbers possible to end the number, a 3 and two 7. So if we pick one of those 3, then there are $\frac{6!}{2!2!}$ ways to arrange the others, so $3 \times \frac{6!}{2!2!}$

$\therefore$ P(number formed is odd) $= \dfrac{\frac{3\times 6!}{2!2!}}{1260} = \dfrac{3}{7}$

c) The 1st number must be 4, 6, 7 or 7. Then we arrange the other numbers. So $4 \times \frac{6!}{2!2!}$ $\therefore$ P(number > 4M) $= \dfrac{\frac{4\times 6!}{2!2!}}{1260} = \dfrac{4}{7}$

d) The 1st number must be 2, 2, 3, 4 or 6. Then we arrange the other numbers (6), so $\frac{5\times 6!}{2!\,2!} = 900$

P(number less than 7M) $= \dfrac{900}{1260} = 5/7$

7 9

22 A four-person team is chosen at random from seven women and nine men.

(a) In how many ways can the team be chosen?

(b) What is the probability that the team will consist of four men?

a) $^{16}C_4 = 1820$

b) $\dfrac{^{9}C_4}{^{16}C_4} = \dfrac{9}{130}$

23 Mr and Mrs Zeno and their four children go to the theatre. They are randomly allocated six adjacent seats in a single row. What is the probability that Mr and Mrs Zeno are allocated seats next to each other?

5 positions possible for the 1st parent $\square\square\square\square\square\square$

then the other parent sits next to her or him. (2 possibilities)

then we arrange the kids (4!)

So $\dfrac{2 \times 5 \times 4!}{6!} = \dfrac{2 \times 5!}{6!} = \dfrac{2}{6} = \dfrac{1}{3}$

From a group of 9 people, how many ways to choose 1 umpire and two teams of 4?

9 possible choices for the umpire.

Then, we choose 4 people within 8 left, so $^{8}C_4 = 70$

So that's $9 \times {}^{8}C_4 = 630$.

BUT we need to divide by 2, as if the 8 people are named A, B, C, D, E, F, G, H, the couple of teams ABCD / EFGH is the same as EFGH / ABCD. So the total number of ways is $\dfrac{630}{2} = 315$