EXPONENTIALS AND LOGARITHMS - CHAPTER REVIEW

1 Find the values of x for which the following are true.

(a) $7^{x+2} = 343$ (b) $4^{x-2} < 128$ (c) $3^x \geq 12$

a) $\Leftrightarrow 7^{x+2} = 7^3$ so $x+2=3$ so $x=1$

b) $128 = 2^7$ so the inequality becomes : $4^{x-2} < 2^7$

$\Leftrightarrow (2^2)^{x-2} < 2^7 \Leftrightarrow 2^{2x-4} < 2^7$ so $2x-4 < 7$

$\Leftrightarrow 2x < 11$ so $x < 11/2$

c) $\Leftrightarrow \ln 3^x \geqslant \ln 12 \Leftrightarrow x \ln 3 \geqslant \ln 12$

$\Leftrightarrow x \geqslant \dfrac{\ln(12)}{\ln 3}$

2 Simplify:

(a) $\log_3 18 + 2\log_3 9 - \log_3 54$ (b) $\log_a(xy^2) + \log_a(yz^2) - \log_a(xz^2)$

a) $\log_3 18 + 2\log_3 9 - \log_3 54 = \log_3 18 + \log_3 9^2 - \log_3 54$

$= \log_3 \dfrac{18 \times 9^2}{54} = \log_3 27 = 3$

b) $\log_a(xy^2) + \log_a(yz^2) - \log_a(xz^2) = \log_a \left[\dfrac{xy^2 \times yz^2}{xz^2}\right]$

$= \log_a \left[y^3\right]$

$= 3\log_a y$

2 Simplify: **(c)** $\log_{10}\frac{6+4\sqrt{6}}{5}+\log_{10}\frac{2\sqrt{6}-3}{2}$

$$\log_{10}\frac{6+4\sqrt{6}}{5}+\log_{10}\frac{2\sqrt{6}-3}{2}=\log_{10}\left[\left(\frac{6+4\sqrt{6}}{5}\right)\left(\frac{2\sqrt{6}-3}{2}\right)\right]$$

$$=\log_{10}\left[\frac{12\sqrt{6}-18+8\times 6-12\sqrt{6}}{10}\right]$$

$$=\log_{10}\left[\frac{30}{10}\right]$$

$$=\log_{10}3$$

(d) $2\log(x+1)-\log(x-1)-2\log(y+1)+\log(y-1)$, given $x=5$ and $y=2$

$$S=2\log(x+1)-\log(x-1)-2\log(y+1)+\log(y-1)$$

$$S=2\log 6-\log 4-2\log 3+\log 1$$

$$S=\log 6^2-\log 4-\log 3^2+0$$

$$S=\log 36-\log 4-\log 9$$

$$S=\log\left[\frac{36}{4\times 9}\right]$$

$$S=\log\left[\frac{36}{36}\right]=\log 1=0$$

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3 Express y in terms of x a) $log_a y = x$ b) $log_{10} y = 2 + log_{10} x - log_{10} x^2$

a) $y = a^x$

b) $\Leftrightarrow \log_{10} y = \log_{10} 100 + \log_{10} \frac{x}{x^2} = \log_{10} 100 + \log_{10} \frac{1}{x}$

So $\log_{10} y = \log_{10} \frac{100}{x}$

so $y = \frac{100}{x}$

7 Use the change of base rule to give each expression as a single term involving natural logarithms.

(a) $\log_{10} 5$ **(b)** $3 + \log_3 6$

a) $\log_{10} 5 = \frac{\ln 5}{\ln 10}$

b) $3 + \log_3 6 = \log_3 27 + \log_3 6 = \log_3 162$

So $3 + \log_3 6 = \frac{\ln 162}{\ln 3}$

8 Solve for x:

(a) $\log_e(x+2) = \log_e(2x)$ **(b)** $\log_e(2x+3) = \log_e x^2$ **(c)** $\log_e x^2 = \log_e\left(\frac{x}{3}\right)$

a) $\Leftrightarrow \ln(x+2) = \ln 2x$

so $x+2 = 2x$ so $2 = x$ or $x = 2$

b) $2x + 3 = x^2 \Leftrightarrow x^2 - 2x - 3 = 0$

$\Delta = (-2)^2 - 4 \times (-3) \times 1 = 4 + 12 = 16 = 4^2$

So $x = \frac{2+4}{2} = \frac{6}{2} = 3$ or $x = \frac{2-4}{2} = -\frac{2}{2} = -1$

Both solutions are possible.

c) so $x^2 = \frac{x}{3} \Leftrightarrow x^2 - \frac{x}{3} = 0$

$\Leftrightarrow x\left[x - \frac{1}{3}\right] = 0$

so either $x = 0$ or $x = \frac{1}{3}$

however $x = 0$ is not possible as we cannot calculate $\ln x$ when $x = 0$

9 Solve for x: (a) $2e^{x} = e^{3x}$ (b) $2e^{x+1} = e^{2x}$ (c) $2e^{-x} = e^{3x-4}$

a) $\Leftrightarrow 2e^{x} - (e^{x})^{3} = 0 \Leftrightarrow e^{x}\left[2 - (e^{x})^{2}\right] = 0$

so either $e^{x} = 0$ which is impossible as $e^{x} > 0$

or $2 - (e^{x})^{2} = 0 \Leftrightarrow (e^{x})^{2} = 2 \Leftrightarrow e^{x} = \pm\sqrt{2}$

But the negative solution is impossible as $e^{x} > 0$ so $e^{x} = \sqrt{2}$

so $\ln e^{x} = \ln\sqrt{2}$ or $x = \ln\sqrt{2} = \ln 2^{1/2} = \frac{1}{2}\ln 2$

b) $\Leftrightarrow 2(e^{x} \times e) = (e^{x})^{2}$ we do a change of variable $X = e^{x}$

$\Leftrightarrow 2eX = X^{2} \Leftrightarrow X^{2} - 2eX = 0$

$\Leftrightarrow X\left[X - 2e\right] = 0$ so either $X = 0$ or $X = 2e$

i.e $e^{x} = 0$ or $e^{x} = 2e$

$e^{x} = 0$ is impossible as $e^{x} > 0$

so $e^{x} = 2e \Rightarrow \ln e^{x} = \ln 2e$ so $x = \ln 2e$

or $x = \ln 2 + \ln e = \ln 2 + 1$

c) $\Leftrightarrow \frac{2}{e^{x}} = \frac{e^{3x}}{e^{4}} \Leftrightarrow 2e^{4} = e^{3x} \times e^{x} = e^{3x+x} = e^{4x}$

so $e^{4x} = 2e^{4} \Leftrightarrow \ln e^{4x} = \ln(2e^{4})$

$4x = \ln(2e^{4}) \Leftrightarrow 4x = \ln 2 + \ln e^{4} = \ln 2 + 4$

So $x = \frac{\ln 2}{4} + 1$

10 \$4000 is invested at 3% p.a. compound interest. How long does it take for this money to:
(a) double in value **(b)** grow to \$10000 **(c)** grow to \$80000?

a) Formula is $P = A(1+0.03)^n = 4{,}000 \times 1.03^n$

For $P = 8000$, we must have $8{,}000 = 4{,}000 \times 1.03^n$

$\Leftrightarrow 1.03^n = 2$ so $n = \dfrac{\ln 2}{\ln 1.03} \simeq 23.4$ periods.

b) $P = 10{,}000$ so $10{,}000 = 4{,}000 \times 1.03^n$

$\Leftrightarrow 1.03^n = \dfrac{10}{4} = 2.5$ so $\ln 1.03^n = \ln 2.5$

or $n \times \ln 1.03 = \ln 2.5$ so $n = \dfrac{\ln 2.5}{\ln 1.03} \simeq 31$ periods.

c) $P = 80{,}000$ so $80{,}000 = 4{,}000 \times 1.03^n$

so $1.03^n = 20$ so $\ln 1.03^n = \ln 20$

$\Leftrightarrow n \times \ln 1.03 = \ln 20$

so $n = \dfrac{\ln 20}{\ln 1.03} \simeq 101$ periods.