SCALAR PRODUCT OF VECTORS IN THREE DIMENSIONS

1 For each of the following pairs of vectors, find **(i)** $\underset{\sim}{a} \propto \underset{\sim}{b}$ **(ii)** the angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

(c) $\underset{\sim}{a} = 4\underset{\sim}{i} - 5\underset{\sim}{j} + 7\underset{\sim}{k}, \underset{\sim}{b} = 2\underset{\sim}{i} + \underset{\sim}{j} + 3\underset{\sim}{k}$

(d) $\underset{\sim}{a} = 6\underset{\sim}{i} - \underset{\sim}{j}, \underset{\sim}{b} = 2\underset{\sim}{j} - \underset{\sim}{k}$

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2 A, B, C, D are four points in space with respective coordinates $(0, 0, 0)$, $(1, 2, 3)$, $(-3, 4, 6)$, $(2, -6, -4)$.

Find:

(a) the position vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$

(b) the magnitude of the angle between $\overrightarrow{AB}$ and $\overrightarrow{CD}$

(c) the position vectors $\overrightarrow{BC}$ and $\overrightarrow{AD}$

(d) the magnitude of the angle between $\overrightarrow{BC}$ and $\overrightarrow{AD}$

(e) the scalar projection of $\overrightarrow{AB}$ on $\overrightarrow{CD}$

(f) the scalar projection of $\overrightarrow{CD}$ on $\overrightarrow{AB}$.

5 If $\underset{\sim}{a} = \underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}$, $\underset{\sim}{b} = 2\underset{\sim}{i} - 3\underset{\sim}{j} + 4\underset{\sim}{k}$ and $\underset{\sim}{c} = -2\underset{\sim}{i} - \underset{\sim}{j} + 3\underset{\sim}{k}$, find:

(a) $(\underset{\sim}{c} - \underset{\sim}{a}) \bullet \underset{\sim}{b}$ **(b)** $(\underset{\sim}{a} + \underset{\sim}{b}) \bullet \underset{\sim}{c}$ **(c)** the scalar projection of $\underset{\sim}{a}$ onto $\underset{\sim}{b}$.

6 If $\underset{\sim}{a} = 3\underset{\sim}{i} + 6\underset{\sim}{j} + 2\underset{\sim}{k}$ and $\underset{\sim}{b} = -6\underset{\sim}{i} + 2\underset{\sim}{j} + 3\underset{\sim}{k}$, find:

(a) a unit vector parallel to $2\underset{\sim}{a} + \underset{\sim}{b}$

(b) a unit vector perpendicular to both $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

7 If $|\underset{\sim}{a}| = |\underset{\sim}{b}|$, simplify the following expressions.

(a) $(\underset{\sim}{a} + \underset{\sim}{b}) \bullet (\underset{\sim}{a} - \underset{\sim}{b})$

(b) $(\underset{\sim}{a} + 2\underset{\sim}{b}) \bullet (2\underset{\sim}{a} - b)$

11 If $\underset{\sim}{u} = \underset{\sim}{i} + 2\underset{\sim}{j} - 2\underset{\sim}{k}$ and $\underset{\sim}{v} = 2\underset{\sim}{i} + 3\underset{\sim}{j} - 6\underset{\sim}{k}$, find:

(a) $\hat{\underset{\sim}{u}}$ **(b)** $\hat{\underset{\sim}{v}}$

(c) a unit vector in the direction $2\underset{\sim}{u} - \underset{\sim}{v}$

(d) **(i)** the vector projection of $\underset{\sim}{u}$ parallel to $\underset{\sim}{v}$

(ii) the vector projections of $\underset{\sim}{u}$ perpendicular to $\underset{\sim}{v}$.

13 Find the value of p for which $\underset{\sim}{i} - 2p\underset{\sim}{j} + 3\underset{\sim}{k}$ and $p\underset{\sim}{i} - 4\underset{\sim}{j} + 3\underset{\sim}{k}$ are perpendicular.

15 If $\underset{\sim}{a} = (6, -2, 6)$ and $\underset{\sim}{b} = (-6, -2, 1)$, find:

(a) $|\underset{\sim}{a}|$

(b) the scalar projection of $\underset{\sim}{b}$ on to $\underset{\sim}{a}$

(c) $|\underset{\sim}{a} - \underset{\sim}{b}|$

(d) the magnitude of the projection of $\underset{\sim}{b}$ on to $\underset{\sim}{a} - \underset{\sim}{b}$.

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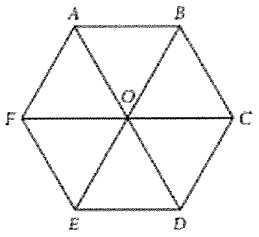
17 $ABCDEF$ is a regular hexagon with centre O. If $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OB} = \underset{\sim}{b}$, express each of the following in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$:



(a) $\overrightarrow{AB}$ (b) $\overrightarrow{BC}$

(c) $\overrightarrow{CD}$ (d) $\overrightarrow{BD}$

(e) $\overrightarrow{FC}$ (f) Prove that $\overrightarrow{BD}$ and $\overrightarrow{FC}$ are perpendicular.

18 Given $\underline{a} = 2\underline{i} - \underline{j} + 2\underline{k}$ and $\underline{b} = \underline{i} + 2\underline{j} - 2\underline{k}$, find two vectors $\underline{c}$ and $\underline{d}$ such that $\underline{a} = \underline{c} + \underline{d}$, $\underline{c}$ is parallel to $\underline{b}$, $\underline{d}$ is perpendicular to $\underline{b}$.

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22 $ABCD$ is a rectangle with vector $\overrightarrow{AB} = 3\underset{\sim}{i}$ and vector $\overrightarrow{AD} = 2\underset{\sim}{j}$.

(a) Express the diagonal vectors $\overrightarrow{AC}$ and $\overrightarrow{DB}$ in terms of $\underset{\sim}{i}$ and $\underset{\sim}{j}$.

(b) Calculate, to the nearest degree, the angle between the diagonals.

23 If $\underset{\sim}{u}$ and $\underset{\sim}{v}$ are vectors defined by $\underset{\sim}{u} = \underset{\sim}{i} + \underset{\sim}{j} + \sqrt{2}\underset{\sim}{k}$ and $\underset{\sim}{v} = \underset{\sim}{i} - \underset{\sim}{j} + \sqrt{2}\underset{\sim}{k}$, find:

(a) a unit vector parallel to $\underset{\sim}{u}$

(b) the angle between $\underset{\sim}{u}$ and $\underset{\sim}{v}$

(c) the vector projection of $\underset{\sim}{v}$ in the direction of $\underset{\sim}{u}$.