GRADIENT OF A STRAIGHT LINE

1 Find the gradient of the line containing the given points.

(a) $(2,4), (0,6)$ **(b)** $(-3,-1), (-5,6)$ **(c)** $(-2,2), (-6,2)$ **(d)** $(2,-3), (-3,2)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$ a) $m_a = \frac{6-4}{0-2} = -1$

b) $m_b = \frac{6-(-1)}{-5-(-3)} = \frac{7}{-2} = -\frac{7}{2}$

c) $m_c = \frac{2-2}{-6-(-2)} = 0$ (horizontal line)

d) $m_d = \frac{2-(-3)}{-3-2} = \frac{5}{-5} = -1$

2 Calculate the angle of inclination of the line joining the given points.

(a) $(-4,-2), (4,6)$ **(b)** $(0,5), (-2,4)$ **(c)** $(-5,6), (3,3)$

a) $m_a = \frac{6-(-2)}{4-(-4)} = \frac{8}{8} = 1$ so $45°$

b) $m_b = \frac{4-5}{-2-0} = \frac{-1}{-2} = \frac{1}{2}$ so $\tan\theta = \frac{1}{2}$ $\theta \simeq 26°34'$

c) $m_c = \frac{3-6}{3-(-5)} = \frac{-3}{8} = \tan\theta$

so $\theta \simeq -20°33'$

3 Calculate the gradients of the lines joining the points $(-2,3)$ and $(4,-2)$ and the points $(-1,7)$ and $(-7,12)$. These two lines are:

$\boxed{\textbf{A}}$ parallel **B** perpendicular **C** intersecting **D** coincident

$m_1 = \frac{-2-3}{4-(-2)} = \frac{-5}{6}$ $m_2 = \frac{12-7}{-7-(-1)} = \frac{5}{-6} = -\frac{5}{6}$

equation is $y - 3 = -\frac{5}{6}(x+2)$ equation is $y - 7 = \frac{-5}{6}(x+1)$

$\Leftrightarrow y = {}^{6}5/6\, x + 4/3$ $\Leftrightarrow y = -5/6\, x + 37/6$

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4 For each of the following, show that $ABCD$ is a parallelogram.

(a) $A(0,0)$, $B(3,0)$, $C(5,5)$, $D(2,5)$ (b) $A(-3,-1)$, $B(4,1)$, $C(8,5)$, $D(1,3)$

a) $m_{AB} = \frac{0-0}{3-0} = 0$ $m_{CD} = \frac{5-5}{2-5} = 0$ so $[AB] // [CD]$

$m_{BC} = \frac{5-0}{5-3} = \frac{5}{2}$ $m_{DA} = \frac{5-0}{2-0} = \frac{5}{2}$ so $[BC] // [DA]$

$\therefore$ ABCD is a //gram.

b) $m_{AB} = \frac{1-(-1)}{4-(-3)} = \frac{2}{7}$ whereas $m_{CD} = \frac{3-5}{1-8} = \frac{-2}{-7} = \frac{2}{7}$

so $[AB] // [CD]$

$m_{BC} = \frac{5-1}{8-4} = \frac{4}{4} = 1$ whereas $m_{DA} = \frac{3+1}{1-(-3)} = \frac{4}{4} = 1$

so $[BC] // [DA]$

$\therefore$ ABCD is a //gram.

6 Show that the points $\overset{A}{(-2,0)}$, $\overset{B}{(2,12)}$ and $\overset{C}{(-5,-9)}$ are collinear.

$m_{AB} = \frac{12-0}{2-(-2)} = \frac{12}{4} = 3$

whereas $m_{BC} = \frac{-9-12}{-5-2} = \frac{-21}{-7} = 3$

$\therefore$ (AB) and (BC) have the same gradient

and have one point in common, which is B.

$\therefore$ Points A, B and C are colinear

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7 For the points $A(2a,b)$, $B(a,2b)$ and $C(-a,4b)$, indicate whether each statement is correct or incorrect.

(a) $AB = BC$ **(b)** $AB + BC = AC$ **(c)** $AB \perp BC$ **(d)** A, B and C are collinear

a) $AB = \sqrt{(2b-b)^2+(a-2a)^2} = \sqrt{b^2+a^2}$

whereas $BC = \sqrt{(4b-2b)^2+(-a-a)^2} = \sqrt{4b^2+4a^2} = 2\sqrt{b^2+a^2} = 2.AB$ so wrong

b) $AC = \sqrt{(4b-b)^2+(-a-2a)^2} = \sqrt{(3b)^2+(3a)^2} = 3\sqrt{b^2+a^2}$

so $AC = BC + AB$ True

c) $m_{AB} = \dfrac{2b-b}{a-2a} = \dfrac{b}{-a} = -\dfrac{b}{a}$ whereas $m_{BC} = \dfrac{4b-2b}{-a-a} = \dfrac{2b}{-2a} = -\dfrac{b}{a}$

so $(AB) \parallel (BC)$ (not $\perp$) wrong.

d) True as shown above.

8 For each of the following, show that ABC is a right-angled triangle.

(a) $A(2,-3)$, $B(5,2)$, $C(-3,0)$ **(b)** $A(-1,2)$, $B(3,4)$, $C(7,-4)$

a) $AB^2 = (2-(-3))^2+(5-2)^2 = 5^2+3^2 = 25+9 = 34$

Method 1 with Pythagoras.

$BC^2 = (0-2)^2+(-3-5)^2 = 4+64 = 68$

$AC^2 = (0-3)^2+(-3-2)^2 = 9+5^2 = 34$

$AB^2+AC^2 = BC^2$ $\therefore$ ABC is a right-angled triangle.

b) Method 2, with gradients.

$m_{AB} = \dfrac{4-2}{3-(-1)} = \dfrac{2}{4} = \dfrac{1}{2}$ $m_{BC} = \dfrac{-4-4}{7-3} = \dfrac{-8}{4} = -2$

$m_{AB} \times m_{BC} = -1$ $\therefore$ ABC is a right angled triangle