THE BINOMIAL DISTRIBUTION - CHAPTER REVIEW

1 Find the stated probability for the following binomial distributions. Express your answers as fractions in simplest form.

(a) $P(X = 3)$ if $X \sim B\left(5, \frac{3}{5}\right)$. **(b)** $P(X = 4)$ if $X \sim B\left(6, \frac{7}{10}\right)$.

a) $P(X=3) = {}^5C_3 \left(\frac{3}{5}\right)^3 \left(1-\frac{3}{5}\right)^{5-3} = \frac{5!}{3!(5-3)!} \times \frac{3^3}{5^3} \times \left(\frac{2}{5}\right)^2 = \frac{216}{625}$

b) $P(X=4) = {}^6C_4 \left(\frac{7}{10}\right)^4 \left(1-\frac{7}{10}\right)^{6-4} = {}^6C_4 (0.7)^4 (0.3)^2 = 0.324135$

2 For the variable $Y \sim B(n, p)$ it is known that $E(Y) = 32$ and $\mathrm{Var}(Y) = 6.4$.

(a) Find the probability of success, p. **(b)** Find the number of trials, n.

$E(Y) = np = 32$ $\quad$ $Var(Y) = np(1-p) = 6.4$ $\quad$ so $1-p = \frac{6.4}{32} = \frac{1}{5}$

so $p = 4/5 = 0.8$

and $n \times p = 32$ so $n = \frac{32}{0.8} = 40$

3 A coin is biased in such a way that $P(\text{heads}) = 3 \times P(\text{tails})$. The coin is tossed 100 times. Let X stand for the number of tails obtained. Find the value of $E(X)$.

$E(X) = np = 100 \times \frac{1}{4} = 25$ $\quad$ ($p = \frac{1}{4}$ as $P(\text{heads}) = 3 \times P(\text{tails})$ and $P(\text{heads}) + P(\text{tails}) = 1$)

4 When Yehudi and Carlos play racquetball, the probability that Yehudi wins a point is 0.35.

(a) How many points would you expect Yehudi to win from the first 15 points? Give your answer to the nearest whole number of points.

(b) Choose the correct terms in the following statement.
If Yehudi won 10 out of the first 15 points, I would [not be / be slightly / be very] surprised as the number is [about the same as / just above / well above] the expected number.

a) $E(X) = 15 \times 0.35 = 5.25$ so 5 points.

b) If Yehudi won 10 out of the first 15 points, I would be very surprised as the number is well above the expected number.

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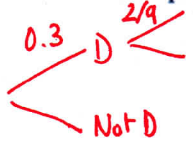
5 A sample of three items is selected at random from a box containing 10 items of which three are defective. Let Y represent the number of defective items selected.

(a) Complete the table to show the probability distribution of the variable. State the probabilities in simplest fraction form.



y	0	1	2	3
$P(Y=y)$	7/24	21/40	7/40	1/120

(b) Find the expected number of defective items, $E(Y)$.

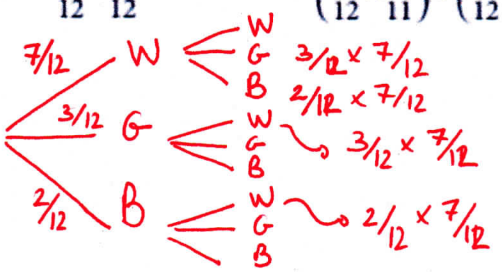
$E(Y) = \sum_{i=0}^{3} i\, p(Y=i) = 0 \times \frac{7}{24} + 1 \times \frac{21}{40} + 2 \times \frac{7}{40} + 3 \times \frac{1}{120}$

$E(Y) = 9/10$

A

6 A jar contains seven white marbles, three green marbles and two blue marbles. Two marbles are drawn, with replacement, from the jar. What is the probability of drawing exactly one white marble?

A $\frac{7}{12} \times \frac{7}{12}$ **B** $\left(\frac{7}{12} \times \frac{5}{11}\right) + \left(\frac{5}{12} \times \frac{7}{11}\right)$ $\boxed{\textbf{C}}$ $2 \times \frac{7}{12} \times \frac{5}{12}$ **D** $\frac{7}{12} \times \frac{5}{12}$



So $\frac{3}{12} \times \frac{7}{12} + \frac{2}{12} \times \frac{7}{12} + \frac{3}{12} \times \frac{7}{12} + \frac{2}{12} \times \frac{7}{12}$

$= 2 \times \frac{7}{12} \times \frac{5}{12}$

7 Find the value of t in the following probability distribution table.

x	0	1	2	3
$P(X=x)$	t	$2t$	$3t$	$4t$

A 10 $\boxed{\textbf{B}}$ 0.1 **C** 6 **D** 0.6

$t + 2t + 3t + 4t = 1$ so $10t = 1$ $t = 0.1$

8 A coin, which is biased so that $P(\text{heads}) = 2 \times P(\text{tails})$, is tossed eight times. The probability that the result is heads exactly three times, $P(X=3)$, is best represented by:

A $\binom{8}{3}\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right)^3$ $\boxed{\textbf{B}}$ $\binom{8}{3}\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^5$ **C** $\binom{8}{5}\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^2$ **D** $\binom{8}{3}\left(\frac{2}{5}\right)^3 + \binom{8}{5}\left(\frac{1}{3}\right)^5$

$P(X=3) = {}^8C_3 \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^{8-3} = {}^8C_3 \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^5$

9 If $Y \sim B(100, 0.2)$, what is the value of μ?

A 0.8 **B** 16 $\boxed{\textbf{C}}$ 20 **D** 80

$\mu = np = 100 \times 0.2 = 20$

10 For a particular binomial random variable Y, it is known that $\text{Var}(Y) = 14.4$. If 60 trials are conducted, what is the probability p of success?

A 0.4 **B** 0.6 **C** 0.24 or 0.76 $\boxed{\textbf{D}}$ 0.4 or 0.6

$\text{Var}(Y) = np(1-p)$ so $14.4 = 60p(1-p)$

$\Leftrightarrow 60p^2 - 60p + 14.4 = 0$ $\Delta = 144 = 12^2$ so $p = \frac{60 \pm 12}{120}$

$p = 0.6$ or $p = 0.4$

11 A die, with sides labelled 1–6, is biased so that $P(\text{odd}) = 3 \times P(\text{even})$.

(a) If rolling an odd number is considered a success, find $P(\text{success})$.

(b) The die is rolled 30 times.

- **(i)** Write this information in the form $X \sim B(n, p)$.
- **(ii)** What is the expected number of odd numbers that will occur in the 30 rolls?
- **(iii)** How unusual would you consider it to roll 28 odd numbers in the 30 rolls? Explain with reference to the 95% confidence interval.

(c) **(i)** What is the probability of any pair of rolls resulting in two odd numbers?

- **(ii)** Draw a table to show the probability distribution of the number of odd numbers in the two rolls.

a) $P(\text{odd}) + P(\text{even}) = 1$ so $4P(\text{even}) = 1$ so $P(\text{even}) = 1/4$ and $P(\text{odd}) = 3/4$

So $P(\text{success}) = 3/4$

b) i) $X \sim B(30, 3/4)$ ii) $E(\text{odd}) = 30 \times \frac{3}{4} = 22.5$

iii) $\text{Var}(X) = 30 \times 0.75 \times 0.25 = 5.625$ so $\sigma = 2.4$

So 95% of results would be between 17.7 and 27.3

So rolling 28 odd would be outside the 95% confidence level.

c) i) $P(2 \text{ odd numbers}) = {}^2C_2 \times \left(\frac{3}{4}\right)^2 \times \left(\frac{1}{4}\right)^0 = 0.75^2 = 0.5625$

ii)

y	0	1	2
$P(Y=y)$	0.0625	0.375	0.5625

$P(Y=0) = {}^2C_0 \times (0.75)^0 \times (0.25)^2$

$P(Y=0) = 0.0625$

$P(Y=1) = {}^2C_1 \times (0.75)^1 \times (0.25)^1$

$P(Y=1) = 0.375$

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12 A cereal manufacturer is running a promotion on single-serving boxes of cereal, which states that one in five cereal boxes contains a free gift. Carol likes to win free gifts and intends to buy one cereal box every day for one week. State your answers correct to three decimal places where necessary.

(a) If the variable X represents the number of cereal boxes with a free gift, calculate the probability that Carol will win exactly one free gift from the seven cereal boxes she intends to buy.

(b) What is the probability that Carol wins no free gifts from the seven cereal boxes she intends to buy?

(c) Given that Carol wins one free gift on the first day, calculate the probability that she wins exactly one more free gift in the next six days.

(d) Given that Carol does not win a free gift in the first five days, calculate the probability that she wins more than one free gift in the next two days.

(e) Carol thinks that if she buys one cereal box per day for 10 days, her chances of winning more than two free gifts will increase. Explain why Carol is correct in her thinking, using appropriate calculations.

a) $P(X=1) = {}^7C_1 \left(\frac{1}{5}\right)^1 \times \left(\frac{4}{5}\right)^{7-1} = {}^7C_1 \left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^6 = 0.367$

b) $P(X=0) = {}^7C_0 \left(\frac{1}{5}\right)^0 \left(\frac{4}{5}\right)^7 = 0.210$

c) $P(X=1) = {}^6C_1 \left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^{6-1} = {}^6C_1 \left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^5 = 0.393$

d) $P(X=2) = \frac{1}{5} \times \frac{1}{5} = \frac{1}{25} = 0.04$

e) Chance of winning more than two if buying one box per day for 7 days is $1 - 0.367 - 0.210 - {}^7C_2 \left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^5$ which is 0.148

Chance of winning more than two boxes if buying one box per day for 10 days is $1 - \underbrace{{}^{10}C_0 \left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^{10}}_{=0.107} - \underbrace{{}^{10}C_1 \left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^9}_{=0.268} - \underbrace{{}^{10}C_2 \left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^8}_{=0.302}$

which is 0.323

Therefore Carol is correct, her chances will increase.