RATE OF CHANGE WITH RESPECT TO TIME

The gradient as a rate of change was introduced in the Mathematics Advanced course. This chapter will extend that topic.

There are many practical situations in which the change in a physical quantity can be described mathematically. The speed of a car depends on the rate at which the distance it travels changes with respect to time. The rate of change in the population of Australia depends on the number of people alive at a given time, together with the birth rate, the death rate and the immigration/emigration rate.

You have seen that the rate of change of a function is the derivative of that function. Hence if you have information about a real-life situation expressed as a function of time, then the rate of change of this can be calculated by finding the derivative of this function.

Example 1

The volume of water in a cylindrical tank of constant cross-section is given by $V = 25\pi(100 - t)\text{ m}^3$, where t is the time in minutes.

(a) How much water is in the tank initially?

(b) If $\frac{dV}{dt}$ gives the rate at which water flows out from the bottom of the tank when a tap is opened, find $\frac{dV}{dt}$ and comment on your answer.

(c) How much time does it take for the tank to empty?

Solution

(a) $t = 0$: $V = 25\pi \times 100 = 2500\pi\text{m}^3$

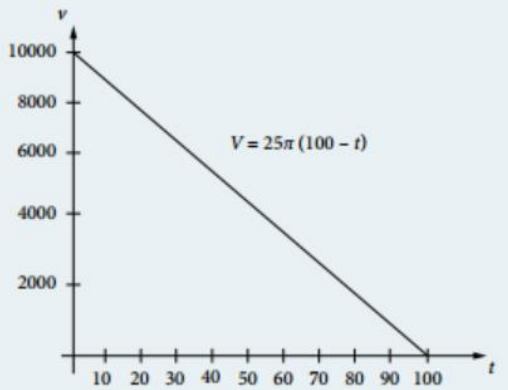
(b) $\frac{dV}{dt} = -25\pi$

The water is flowing out of the tank at a constant rate.

(c) The tank is empty when $V = 0$: $25\pi(100 - t) = 0$

$t = 100$ minutes

This could also be solved by considering where the graph of $V = 25\pi(100 - t)$ cuts the t-axis.



The graph cuts the t-axis (that is, reaches $V = 0$) at $t = 100$, so the tank is empty after 100 minutes.

Example 2

A large cube of ice has edges of length 10 cm. As it melts its volume decreases at a constant rate of $25\,cm^3$ per hour.

(a) What is the initial volume of the cube of ice?

(b) What is the volume of ice remaining after 2 hours?

(c) Obtain a formula for the volume $V\,cm^3$ remaining after t hours.

(d) How much time will it take for the ice to melt completely?

Solution

(a) Volume of cube of ice $= 10^3 = 1000\,cm^3$.

(b) After 2 hours, the volume melted $= 25 \times 2 = 50\,cm^3$.
Volume remaining $= 1000 - 50 = 950\,cm^3$.

(c) After t hours, the volume melted $= 25 \times t = 25t\,cm^3$.
$V = 1000 - 25t$.

(d) $V = 0$: $1000 - 25t = 0$, so $t = 40$ hours.
It takes 40 hours for the cube of ice to melt completely.