GRAPHICAL SOLUTION OF EQUATIONS

Types of equations

So far we studied / learnt to solve a number of equations:

Example	Type of equation
$4x + 3 = 0$	linear
$2x^2 - 4x + 1 = 0$	quadratic
$5x^3 + 3x^2 - 2x + 1 = 0$	cubic
$\sin x = 0.5, \cos x = 0.3, \tan x = 2$	trigonometric
$6^x = 8$	exponential
$\log_2 x = 3$	logarithmic
$2\frac{d^2 f(x)}{dx^2} + 3\frac{df(x)}{dx} - f(x) = 6$	differential (the solution to this kind of equation is a family of functions, NOT a variable x like for the others kind of equations listed above)

Equations such as $\sin x = 1 - 2x$ or $\cos x = x^2 + 3$

Equations which mix different kinds of components (in both cases above, a trigonometric component and a linear component) are called **transcendental equations**.

Often, it is not possible to find an exact value for the solution of this kind of equations, the best we can achieve is to find an approximate solution.

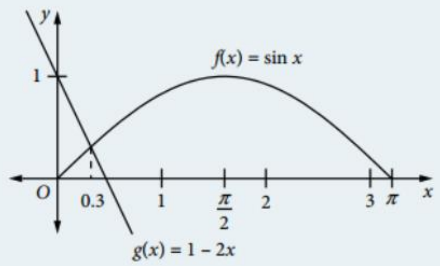
One of the ways to find an approximate solution to an acceptable level of accuracy is through try-and-error, or through a graphical method, as described below:

Example 11

Solve $\sin x = 1 - 2x$.

Solution

Sketch the functions.



This shows that the graphs of $f(x) = \sin x$ and $g(x) = 1 - 2x$ intersect at only one point, approximately $x = 0.3$, so 0.3 is an approximate solution of $\sin x = 1 - 2x$. Testing values of x to refine this approximation:

x	0.3	0.4	0.35	0.335
$\sin x$	0.2955	0.3894	0.3429	0.3288
$1 - 2x$	0.4	0.2	0.3	0.33

This table shows the values of $\sin x$ and $1 - 2x$ for values of x between $x = 0.3$ and $x = 0.4$. $x = 0.35$ is a better solution than $x = 0.3$. Further trial and error leads to a solution of $x = 0.335$.

A 'find the intersection' function in a graphing software program would allow you to find this solution quickly and accurately. Unfortunately, under examination conditions you may have to use the sketching method.