FURTHER WORK WITH VECTORS - CHAPTER REVIEW

1 For each of the points, P, whose coordinates are given, find:

(i) an $\underset{\sim}{i}, \underset{\sim}{j}, \underset{\sim}{k}$ representation for the position vector $\overrightarrow{OP}$ (ii) the magnitude of $\overrightarrow{OP}$

(iii) a unit vector in the direction of $\overrightarrow{OP}$.

(a) $P(-1, 4, 2)$ **(b)** $P(3, 6, 8)$ **(c)** $P(-2, 2, -1)$

a) i) $\overrightarrow{OP} = -\vec{i} + 4\vec{j} + 2\vec{k}$

ii) $|\overrightarrow{OP}| = \sqrt{(-1)^2 + 4^2 + 2^2} = \sqrt{21}$

iii) $\dfrac{\overrightarrow{OP}}{|\overrightarrow{OP}|} = \dfrac{1}{\sqrt{21}}\left[-\vec{i} + 4\vec{j} + 2\vec{k}\right]$

b) $\overrightarrow{OP} = 3\vec{i} + 6\vec{j} + 8\vec{k}$

$|\overrightarrow{OP}| = \sqrt{3^2 + 6^2 + 8^2} = \sqrt{109}$

$\dfrac{\overrightarrow{OP}}{|\overrightarrow{OP}|} = \dfrac{1}{\sqrt{109}}\left[3\vec{i} + 6\vec{j} + 8\vec{k}\right]$

c) $\overrightarrow{OP} = -2\vec{i} + 2\vec{j} - \vec{k}$

$|\overrightarrow{OP}| = \sqrt{(-2)^2 + 2^2 + (-1)^2} = \sqrt{9} = 3$

$\dfrac{\overrightarrow{OP}}{|\overrightarrow{OP}|} = \dfrac{1}{3}\left[-2\vec{i} + 2\vec{j} - \vec{k}\right]$

FURTHER WORK WITH VECTORS - CHAPTER REVIEW

2 Given $A(3, 3, 1)$, $B(-2, 1, -1)$, $C(1, 1, 1)$ and $D(2, 1, -2)$, find:

(a) the angle between $\overrightarrow{AB}$ and $\overrightarrow{CD}$ **(b)** the angle between $\overrightarrow{AC}$ and $\overrightarrow{BD}$
(c) the angle between $\overrightarrow{AD}$ and $\overrightarrow{BC}$.

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -\overrightarrow{OA} + \overrightarrow{OB} = -(3\vec{i} + 3\vec{j} + \vec{k}) + (-2\vec{i} + \vec{j} - \vec{k})$$

$$\overrightarrow{AB} = -5\vec{i} - 2\vec{j} - 2\vec{k}$$

$$\overrightarrow{CD} = -\overrightarrow{OC} + \overrightarrow{OD} = -(\vec{i} + \vec{j} + \vec{k}) + (2\vec{i} + \vec{j} - 2\vec{k}) = \vec{i} - 3\vec{k}$$

$$\overrightarrow{AC} = -\overrightarrow{OA} + \overrightarrow{OC} = -(3\vec{i} + 3\vec{j} + \vec{k}) + (\vec{i} + \vec{j} + \vec{k}) = -2\vec{i} - 2\vec{j}$$

$$\overrightarrow{BD} = -\overrightarrow{OB} + \overrightarrow{OD} = -(-2\vec{i} + \vec{j} - \vec{k}) + (2\vec{i} + \vec{j} - 2\vec{k}) = 4\vec{i} - \vec{k}$$

$$\overrightarrow{AD} = -\overrightarrow{OA} + \overrightarrow{OD} = -(3\vec{i} + 3\vec{j} + \vec{k}) + (2\vec{i} + \vec{j} - 2\vec{k}) = -\vec{i} - 2\vec{j} - 3\vec{k}$$

$$\overrightarrow{BC} = -\overrightarrow{OB} + \overrightarrow{OC} = -(-2\vec{i} + \vec{j} - \vec{k}) + (\vec{i} + \vec{j} + \vec{k}) = 3\vec{i} + 2\vec{k}$$

$$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos\theta \quad \text{so} \quad \cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}$$

a) $$\cos(\overrightarrow{AB}, \overrightarrow{CD}) = \frac{(-5\vec{i} - 2\vec{j} - 2\vec{k}) \cdot (\vec{i} - 3\vec{k})}{\sqrt{5^2 + 2^2 + 2^2}\sqrt{1^2 + 3^2}} = \frac{-5 + 6}{\sqrt{33}\sqrt{10}} = \frac{1}{\sqrt{330}}$$

$$\theta = 86° 51'$$

b) $$\cos(\overrightarrow{AC}, \overrightarrow{BD}) = \frac{(-2\vec{i} - 2\vec{j}) \cdot (4\vec{i} - \vec{k})}{\sqrt{2^2 + 2^2}\sqrt{4^2 + 1^2}} = \frac{-8}{\sqrt{8}\sqrt{17}} = -\frac{8}{\sqrt{136}}$$

$$\theta = 133° 19'$$

c) $$\cos(\overrightarrow{AD}, \overrightarrow{BC}) = \frac{(-\vec{i} - 2\vec{j} - 3\vec{k}) \cdot (3\vec{i} + 2\vec{k})}{\sqrt{1^2 + 2^2 + 3^2}\sqrt{3^2 + 2^2}} = \frac{-3 - 6}{\sqrt{14}\sqrt{13}}$$

so $$\cos(\overrightarrow{AD}, \overrightarrow{BC}) = \frac{-9}{\sqrt{182}} \qquad \theta = 131° 51'$$

FURTHER WORK WITH VECTORS - CHAPTER REVIEW

3 Determine whether the given sets of points are collinear.

(a) $A(1, 3, 2)$, $B(3, 1, 4)$, $C(5, -2, -6)$ **(b)** $D(1, 3, -4)$, $E(3, -2, 2)$, $F(3, 1, 5)$

a) $\vec{AB} = -\vec{OA} + \vec{OB} = -(\vec{i} + 3\vec{j} + 2\vec{k}) + (3\vec{i} + \vec{j} + 4\vec{k}) = 2\vec{i} - 2\vec{j} + 2\vec{k}$

$\vec{BC} = -\vec{OB} + \vec{OC} = -(3\vec{i} + \vec{j} + 4\vec{k}) + (5\vec{i} - 2\vec{j} - 6\vec{k}) = 2\vec{i} - 3\vec{j} - 10\vec{k}$

So $\nexists\, k \in \mathbb{R}$ s.t. $\vec{AB} = k\,\vec{BC}$ therefore A, B, C not collinear

b) $\vec{DE} = -\vec{OD} + \vec{OE} = -(\vec{i} + 3\vec{j} - 4\vec{k}) + (3\vec{i} - 2\vec{j} + 2\vec{k})$

$\vec{DE} = 2\vec{i} - 5\vec{j} + 6\vec{k}$

$\vec{EF} = -\vec{OE} + \vec{OF} = -(3\vec{i} - 2\vec{j} + 2\vec{k}) + (3\vec{i} + \vec{j} + 5\vec{k})$

$\vec{EF} = 3\vec{j} + 3\vec{k}$

So $\nexists\, k \in \mathbb{R}$ s.t. $\vec{DE} = k\,\vec{EF}$ $\therefore$ D, E, F not collinear

4 Given $\underset{\sim}{a} = 2\underset{\sim}{i} + 3\underset{\sim}{j} - 4\underset{\sim}{k}$, $\underset{\sim}{b} = 3\underset{\sim}{i} - 5\underset{\sim}{j} - 4\underset{\sim}{k}$, $\underset{\sim}{c} = 2\underset{\sim}{i} + 6\underset{\sim}{j} + 3\underset{\sim}{k}$, find unit vectors $\hat{\underset{\sim}{a}}$, $\hat{\underset{\sim}{b}}$, $\hat{\underset{\sim}{c}}$.

$|\vec{a}| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29}$ so $\dfrac{\vec{a}}{|\vec{a}|} = \dfrac{1}{\sqrt{29}}\left[2\vec{i} + 3\vec{j} - 4\vec{k}\right]$

$|\vec{b}| = \sqrt{3^2 + 5^2 + 4^2} = \sqrt{50} = 5\sqrt{2}$ so $\dfrac{\vec{b}}{|\vec{b}|} = \dfrac{1}{5\sqrt{2}}\left[3\vec{i} - 5\vec{j} - 4\vec{k}\right]$

$|\vec{c}| = \sqrt{2^2 + 6^2 + 3^2} = \sqrt{49} = 7$ so $\dfrac{\vec{c}}{|\vec{c}|} = \dfrac{1}{7}\left[2\vec{i} + 6\vec{j} + 3\vec{k}\right]$

FURTHER WORK WITH VECTORS - CHAPTER REVIEW

5 If $\underset{\sim}{a} = 2\underset{\sim}{i} + 3\underset{\sim}{j} + 4\underset{\sim}{k}$, $\underset{\sim}{b} = 4\underset{\sim}{i} - \underset{\sim}{j} - 2\underset{\sim}{k}$ and $\underset{\sim}{c} = -5\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$, simplify:

(a) $(\underset{\sim}{a} \bullet \underset{\sim}{b})\underset{\sim}{c} + (\underset{\sim}{a} \bullet \underset{\sim}{c})\underset{\sim}{b}$ **(b)** $(\underset{\sim}{c} - \underset{\sim}{a}) \bullet \underset{\sim}{b}$ **(c)** $(\underset{\sim}{a} - \underset{\sim}{b}) \bullet (\underset{\sim}{b} - \underset{\sim}{c})$

a) $(\vec{a}.\vec{b})\vec{c} + (\vec{a}.\vec{c})\vec{b} = [(2\vec{i}+3\vec{j}+4\vec{k}).(4\vec{i}-\vec{j}-2\vec{k})]\vec{c}$
$+ [(2\vec{i}+3\vec{j}+4\vec{k}).(-5\vec{i}+2\vec{j}-\vec{k})].\vec{b}$

$= [8-3-8]\vec{c} + [-10+6-4]\vec{b} = -3\vec{c} - 8\vec{b}$

$= -3(-5\vec{i}+2\vec{j}-\vec{k}) - 8(4\vec{i}-\vec{j}-2\vec{k})$

$= -17\vec{i} + 2\vec{j} + 19\vec{k}$

b) $(\vec{c}-\vec{a}).\vec{b} = [(-5\vec{i}+2\vec{j}-\vec{k}) - (2\vec{i}+3\vec{j}+4\vec{k})].\vec{b}$

$= [-7\vec{i} - \vec{j} - 5\vec{k}].[4\vec{i} - \vec{j} - 2\vec{k}]$

$= -28 + 1 + 10 = -17$

c) $(\vec{a}-\vec{b}).(\vec{b}-\vec{c}) = [(2\vec{i}+3\vec{j}+4\vec{k}) - (4\vec{i}-\vec{j}-2\vec{k})].$
$[(4\vec{i}-\vec{j}-2\vec{k}) - (-5\vec{i}+2\vec{j}-\vec{k})]$

$= [-2\vec{i} + 4\vec{j} + 6\vec{k}].[9\vec{i} - 3\vec{j} - \vec{k}]$

$= -18 - 12 - 6$

$= -36$

6 The position vectors of the points P, Q and R are $8\underset{\sim}{i} - 4\underset{\sim}{j} - 3\underset{\sim}{k}$, $6\underset{\sim}{i} + 3\underset{\sim}{j} - 4\underset{\sim}{k}$ and $7\underset{\sim}{i} + 5\underset{\sim}{j} - 5\underset{\sim}{k}$ respectively. Find the angle between $\overrightarrow{PQ}$ and $\overrightarrow{QR}$.

$$\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ} = -\overrightarrow{OP} + \overrightarrow{OQ} = -(8\vec{i} - 4\vec{j} - 3\vec{k}) + (6\vec{i} + 3\vec{j} - 4\vec{k})$$

$$\overrightarrow{PQ} = -2\vec{i} + 7\vec{j} - \vec{k}$$

$$\overrightarrow{QR} = -\overrightarrow{OQ} + \overrightarrow{OR} = -(6\vec{i} + 3\vec{j} - 4\vec{k}) + (7\vec{i} + 5\vec{j} - 5\vec{k})$$

$$\overrightarrow{QR} = \vec{i} + 2\vec{j} - \vec{k}$$

$$\cos(\overrightarrow{PQ}, \overrightarrow{QR}) = \frac{\overrightarrow{PQ} \cdot \overrightarrow{QR}}{|\overrightarrow{PQ}||\overrightarrow{QR}|} = \frac{[-2\vec{i} + 7\vec{j} - \vec{k}].[\vec{i} + 2\vec{j} - \vec{k}]}{\sqrt{2^2+7^2+1^2}\sqrt{1^2+2^2+1^2}}$$

$$\cos(\overrightarrow{PQ}, \overrightarrow{QR}) = \frac{-2+14+1}{\sqrt{54}\sqrt{6}}$$

$$\text{———} = \frac{13}{\sqrt{324}}$$

$$\text{so } \widehat{\overrightarrow{PQ}, \overrightarrow{QR}} = \cos^{-1}\left(\frac{13}{\sqrt{324}}\right) = 43^\circ 46'$$

7 Find a vector perpendicular to both $\underset{\sim}{u} = 4\underset{\sim}{i} - 7\underset{\sim}{j} + 4\underset{\sim}{k}$ and $\underset{\sim}{v} = -7\underset{\sim}{i} + 4\underset{\sim}{j} + 4\underset{\sim}{k}$.

Let $\vec{a} = x\vec{i} + y\vec{j} + z\vec{k}$

$\vec{a}.\vec{u} = 0$ and $\vec{a}.\vec{v} = 0$

$\vec{a}.\vec{u} = (x\vec{i} + y\vec{j} + z\vec{k}).(4\vec{i} - 7\vec{j} + 4\vec{k})$

$\quad = 4x - 7y + 4z$ so $4x - 7y + 4z = 0$ ①

$\vec{a}.\vec{v} = (x\vec{i} + y\vec{j} + z\vec{k}).(-7\vec{i} + 4\vec{j} + 4\vec{k})$

$\quad = -7x + 4y + 4z$ so $-7x + 4y + 4z = 0$ ②

Subtracting ① and ②, we obtain $11x - 11y = 0$ so $y = x$

Substituting into ①, we obtain $4x - 7x + 4z = 0$

so $-3x + 4z = 0$ $\quad z = \frac{3}{4}x$

So the vector $\vec{a}$ would be like $x\vec{i} + x\vec{j} + \frac{3}{4}x\vec{k}$

Taking $x = 4$ gives $4\vec{i} + 4\vec{j} + 3\vec{k}$

8 Show that each given equation is the equation of a sphere and find the coordinates of its centre and the radius.

(a) $x^2 + y^2 + z^2 + 14x - 12y + 2z + 5 = 0$ (b) $x^2 + y^2 + z^2 - 6x + 2z + 6 = 0$

a) $\Leftrightarrow x^2 + 14x + y^2 - 12y + z^2 + 2z = -5$

$\Leftrightarrow (x+7)^2 - 49 + (y-6)^2 - 36 + (z+1)^2 - 1 = -5$

$\Leftrightarrow (x+7)^2 + (y-6)^2 + (z+1)^2 = -5 + 1 + 36 + 49$

$\Leftrightarrow$ ______ $= 81 = 9^2$

So sphere of centre $(-7, 6, -1)$ radius 9.

b) $\Leftrightarrow x^2 - 6x + y^2 + z^2 + 2z = -6$

$\Leftrightarrow (x-3)^2 + y^2 + (z+1)^2 = -6 + 9 + 1$

$\Leftrightarrow$ ______ $= 4 = 2^2$

So sphere of centre $(3, 0, -1)$ radius 2

9 For the curves whose parametric equations are given, find:

(i) the Cartesian equation **(ii)** the vector equation.

(a) $x = 2t, y = t^2, t \in R$ **(b)** $x = \sec\theta, y = \tan\theta, -\frac{\pi}{2} < \theta < \frac{\pi}{2}$.

a) i) $\begin{cases} x = 2t \\ y = t^2 \end{cases}$ so $t = x/2$

so $y = \left(\frac{x}{2}\right)^2 = \frac{x^2}{4}$

ii) $\vec{r} = 2t\,\vec{i} + t^2\,\vec{j}$

b) i) $x = \sec\theta = \frac{1}{\cos\theta}$ so $\cos\theta = 1/x$

$y = \tan\theta = \frac{\sin\theta}{\cos\theta} = \sin\theta \times \frac{1}{\cos\theta} = \sqrt{1-\cos^2\theta} \times x$

$y = \sqrt{1 - \frac{1}{x^2}} \times x = \sqrt{x^2 - 1}$ or $y^2 = x^2 - 1$

$\Leftrightarrow x^2 - y^2 = 1$

ii) $\vec{r} = \sec\theta\,\vec{i} + \tan\theta\,\vec{j}$

12 Find the vector equation of the line through $A(4, 3, 6)$ and $B(2, 5, 3)$.

$\vec{r} = \vec{OA} + \lambda \vec{AB} = 4\vec{i} + 3\vec{j} + 6\vec{k} + \lambda[-2\vec{i} + 2\vec{j} - 3\vec{k}]$

So $\vec{r} = [4 - 2\lambda]\vec{i} + [3 + 2\lambda]\vec{j} + [6 - 3\lambda]\vec{k}$

A B

13 Show that the line through the points $(1, -1, 1)$ and $(5, 3, 3)$ is perpendicular to the line through the points $(1, 1, 2)$ and $(4, -4, 6)$.

C D

$\vec{AB} = -\vec{OA} + \vec{OB} = -(\vec{i} - \vec{j} + \vec{k}) + (5\vec{i} + 3\vec{j} + 3\vec{k})$

$\vec{AB} = 4\vec{i} + 4\vec{j} + 2\vec{k}$

$\vec{CD} = -\vec{OC} + \vec{OD} = -(\vec{i} + \vec{j} + 2\vec{k}) + (4\vec{i} - 4\vec{j} + 6\vec{k})$

$\vec{CD} = 3\vec{i} - 5\vec{j} + 4\vec{k}$

$\vec{AB} \cdot \vec{CD} = (4\vec{i} + 4\vec{j} + 2\vec{k}) \cdot (3\vec{i} - 5\vec{j} + 4\vec{k})$

$\_\_\_\_ = 4 \times 3 + 4 \times (-5) + 2 \times 4$

$\_\_\_\_ = 12 - 20 + 8$

$\_\_\_\_ = 0$

$\therefore \vec{AB} \perp \vec{CD}$ The two lines are perpendicular

15 If $\underset{\sim}{a} = \underset{\sim}{i} + 2\underset{\sim}{j} - 3\underset{\sim}{k}$, $\underset{\sim}{b} = 5\underset{\sim}{i} + 2\underset{\sim}{j} - 4\underset{\sim}{k}$, $\underset{\sim}{c} = 2\underset{\sim}{i} - \underset{\sim}{j} - 4\underset{\sim}{k}$, find the values of p and q such that $\underset{\sim}{a} + p\underset{\sim}{b} + q\underset{\sim}{c}$ is parallel to the y-axis.

For $\vec{a} + p\vec{b} + q\vec{c}$ to be parallel to the y-axis, the $\vec{i}$ and $\vec{k}$ components must be zero.

$$\vec{a} + p\vec{b} + q\vec{c} = [\vec{i} + 2\vec{j} - 3\vec{k}] + p[5\vec{i} + 2\vec{j} - 4\vec{k}] + q[2\vec{i} - \vec{j} - 4\vec{k}]$$

$$= [1 + 5p + 2q]\vec{i} + [2 + 2p - 2q]\vec{j} + [-3 - 4p - 4q]\vec{k}$$

So we must have $\begin{cases} 1 + 5p + 2q = 0 \\ -3 - 4p - 4q = 0 \end{cases} \Leftrightarrow \begin{cases} 2 + 10p + 4q = 0 \\ -3 - 4p - 4q = 0 \end{cases}$

By elimination, $-1 + 6p = 0$ so $\boxed{p = 1/6}$

and $\therefore$ $2q = -1 - 5p = -1 - 5 \times 1/6 = -\dfrac{11}{6}$

so $\boxed{q = -\dfrac{11}{12}}$

16 **(a)** Show that the points $O(0, 0, 0)$, $A(1, 1, 0)$, $B(1, 0, 1)$ and $C(0, 1, 1)$ are the vertices of a regular tetrahedron by finding the lengths of each of the six edges.

(b) Use the dot product to find the angle between any two edges.

(c) If M is the midpoint of BC, find the size of $\angle AMB$.

a) $|\vec{OA}| = \sqrt{1^2+1^2+0^2} = \sqrt{2}$ $\quad |\vec{OB}| = \sqrt{1^2+1^2} = \sqrt{2}$

$|\vec{OC}| = \sqrt{1^2+1^2} = \sqrt{2}$ $\quad |\vec{AB}| = \sqrt{1^2+1^2} = \sqrt{2}$

$|\vec{AC}| = \sqrt{1^2+1^2} = \sqrt{2}$ $\quad |\vec{BC}| = \sqrt{1^2+1^2} = \sqrt{2}$

So indeed it's a regular tetrahedron.

b) $\cos\theta = \dfrac{\vec{OA}\cdot\vec{OB}}{|\vec{OA}||\vec{OB}|} = \dfrac{(\vec{i}+\vec{j})\cdot(\vec{i}+\vec{k})}{\sqrt{2}\,\sqrt{2}} = \dfrac{1}{2}$ so $\theta = \frac{\pi}{3}$ radians.

c) $\cos(\vec{AM}, \vec{MB}) = \dfrac{\vec{AM}\cdot\vec{MB}}{|\vec{AM}||\vec{MB}|} = \dfrac{(\vec{AB}+\vec{BM})\cdot\vec{MB}}{|\vec{AM}||\vec{MB}|}$

$\vec{AM} = \vec{AB}+\vec{BM} = \left[-\vec{OA}+\vec{OB}\right] + \frac{1}{2}\vec{BC}$

$\vec{AM} = \left[-(\vec{i}+\vec{j})+(\vec{i}+\vec{k})\right] + \frac{1}{2}\left[-(\vec{i}+\vec{k})+(\vec{j}+\vec{k})\right]$

$\vec{AM} = \left[-\vec{j}+\vec{k}\right] + \frac{1}{2}\left[-\vec{i}+\vec{j}\right] = -\frac{1}{2}\vec{i} - \frac{1}{2}\vec{j} + \vec{k}$

$|\vec{AM}| = \sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2+1^2} = \sqrt{\frac{3}{2}}$

$\cos(\vec{AM}, \vec{MB}) = \dfrac{\left(-\frac{1}{2}\vec{i}-\frac{1}{2}\vec{j}+\vec{k}\right)\cdot\left[-\frac{1}{2}\left(-\vec{i}+\vec{j}\right)\right]}{\sqrt{\frac{3}{2}}\,\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2}}$

$\_\_\_\_ = \dfrac{-1/4 + 1/4}{\sqrt{\frac{3}{2}}\sqrt{\frac{1}{2}}} = 0$ so we must have $\vec{AM} \perp \vec{MB}$

the angle is $\pi/2$ radians.

FURTHER WORK WITH VECTORS - CHAPTER REVIEW

17 Relative to a fixed origin, the points A, B and C are defined respectively by the position vectors $\underset{\sim}{a} = \underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$, $\underset{\sim}{b} = 2\underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{c} = m\underset{\sim}{i}$, where m is a real constant.

(a) If $\angle ABC = \frac{\pi}{3}$, find m. **(b)** If $\angle ABC = \frac{\pi}{2}$, find m.

a) $\cos(\overrightarrow{AB}, \overrightarrow{BC}) = \dfrac{\overrightarrow{AB} \cdot \overrightarrow{BC}}{|\overrightarrow{AB}||\overrightarrow{BC}|} = \cos\frac{\pi}{3} = \frac{1}{2}$

$\overrightarrow{AB} = -\overrightarrow{OA} + \overrightarrow{OB} = -(\vec{i} - \vec{j} + 2\vec{k}) + (2\vec{i} + \vec{j} + \vec{k}) = \vec{i} + 2\vec{j} - \vec{k}$

$|\overrightarrow{AB}| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6}$

$\overrightarrow{BC} = -\overrightarrow{OB} + \overrightarrow{OC} = -(2\vec{i} + \vec{j} + \vec{k}) + m\vec{i} = (m-2)\vec{i} - \vec{j} - \vec{k}$

$|\overrightarrow{BC}| = \sqrt{(m-2)^2 + 1^2 + 1^2} = \sqrt{m^2 - 2m + 6}$

So $\dfrac{1}{2} = \dfrac{(\vec{i} + 2\vec{j} - \vec{k}) \cdot [(m-2)\vec{i} - \vec{j} - \vec{k}]}{\sqrt{6}\sqrt{m^2 - 2m + 6}}$

$\Leftrightarrow \dfrac{1}{2} = \dfrac{(m-2) - 2 + 1}{\sqrt{6}\sqrt{m^2 - 2m + 6}} = \dfrac{m-3}{\sqrt{6}\sqrt{m^2 - 2m + 6}}$

so $6(m^2 - 2m + 6) = 4(m-3)^2 = 4m^2 - 24m + 36$

$\Leftrightarrow 2m^2 + 12m = 0 \Leftrightarrow m(m+6) = 0$

so $m = 0$ or $m = -6$

b) $0 = m - 3$ so $m = 3$

as $\cos\theta = 0$