RATIONALISING DENOMINATORS

For questions **1** to **27**, express each fraction with a rational denominator.

1 $\frac{2}{\sqrt{3}}$ **2** $\frac{\sqrt{5}}{\sqrt{3}}$ **3** $\frac{3\sqrt{5}}{\sqrt{15}}$ **4** $\frac{1}{\sqrt{3}-\sqrt{2}}$

$= \frac{2\sqrt{3}}{\sqrt{3}\sqrt{3}} = \frac{2\sqrt{3}}{3}$ $= \frac{\sqrt{5}\sqrt{3}}{3} = \frac{\sqrt{15}}{3}$ $= \frac{3\sqrt{5}}{\sqrt{3}\sqrt{5}} = \frac{3}{\sqrt{3}} = \sqrt{3}$

(4) $\frac{1}{\sqrt{3}-\sqrt{2}} = \frac{\sqrt{3}+\sqrt{2}}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})} = \frac{\sqrt{3}+\sqrt{2}}{3-2} = \sqrt{3}+\sqrt{2}$

5 $\frac{1}{2\sqrt{7}+\sqrt{6}}$ **6** $\frac{1}{\sqrt{5}+2}$ **7** $\frac{1}{2\sqrt{5}-3\sqrt{2}}$ **8** $\frac{3\sqrt{2}}{\sqrt{5}-\sqrt{3}}$

(5) $\frac{1}{2\sqrt{7}+\sqrt{6}} = \frac{2\sqrt{7}-\sqrt{6}}{(2\sqrt{7}+\sqrt{6})(2\sqrt{7}-\sqrt{6})} = \frac{2\sqrt{7}-\sqrt{6}}{4\times 7-6} = \frac{2\sqrt{7}-\sqrt{6}}{22}$

(6) $\frac{1}{\sqrt{5}+2} = \frac{\sqrt{5}-2}{(\sqrt{5}+2)(\sqrt{5}-2)} = \frac{\sqrt{5}-2}{5-4} = \sqrt{5}-2$

(7) $\frac{1}{2\sqrt{5}-3\sqrt{2}} = \frac{2\sqrt{5}+3\sqrt{2}}{(2\sqrt{5}-3\sqrt{2})(2\sqrt{5}+3\sqrt{2})} = \frac{2\sqrt{5}+3\sqrt{2}}{4\times 5-18} = \frac{2\sqrt{5}+3\sqrt{2}}{2}$

(8) $\frac{3\sqrt{2}}{\sqrt{5}-\sqrt{3}} = \frac{3\sqrt{2}(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})} = \frac{3\sqrt{10}+3\sqrt{6}}{5-3} = \frac{3(\sqrt{10}+\sqrt{6})}{2}$

13 $\frac{\sqrt{7}-2\sqrt{5}}{3\sqrt{5}-2\sqrt{2}}$ **14** $\frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}$ **15** $\frac{5\sqrt{3}+3\sqrt{5}}{5\sqrt{5}-3\sqrt{3}}$ **16** $\frac{2\sqrt{3}}{3\sqrt{3}-2}$

(13) $\frac{\sqrt{7}-2\sqrt{5}}{(3\sqrt{5}-2\sqrt{2})} = \frac{(\sqrt{7}-2\sqrt{5})(3\sqrt{5}+2\sqrt{2})}{(3\sqrt{5}-2\sqrt{2})(3\sqrt{5}+2\sqrt{2})} = \frac{3\sqrt{35}+2\sqrt{14}-30-4\sqrt{10}}{45-8} = \frac{3\sqrt{35}+2\sqrt{14}-30-4\sqrt{10}}{37}$

(14) $\frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = \frac{(3\sqrt{2}+2\sqrt{3})(3\sqrt{2}+2\sqrt{3})}{(3\sqrt{2}-2\sqrt{3})(3\sqrt{2}+2\sqrt{3})} = \frac{18+6\sqrt{6}+6\sqrt{6}+12}{18-12} = \frac{30+12\sqrt{6}}{6} = 5+2\sqrt{6}$

(15) $\frac{(5\sqrt{3}+3\sqrt{5})(5\sqrt{5}+3\sqrt{3})}{(5\sqrt{5}-3\sqrt{3})(5\sqrt{5}+3\sqrt{3})} = \frac{25\sqrt{15}+45+75+9\sqrt{15}}{125-27} = \frac{34\sqrt{15}+120}{98} = \frac{17\sqrt{15}+60}{49}$

(16) $\frac{2\sqrt{3}}{3\sqrt{3}-2} = \frac{2\sqrt{3}(3\sqrt{3}+2)}{(3\sqrt{3}-2)(3\sqrt{3}+2)} = \frac{18+4\sqrt{3}}{27-4} = \frac{18+4\sqrt{3}}{23}$

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21 $\dfrac{\sqrt{3}}{\sqrt{24}-\sqrt{48}}$ **22** $\dfrac{\sqrt{2}-1}{\sqrt{2}+1}$ **23** $\dfrac{2\sqrt{5}-\sqrt{2}}{2\sqrt{5}+\sqrt{2}}$ **24** $\dfrac{\sqrt{6}+2\sqrt{3}}{2\sqrt{6}-\sqrt{3}}$

(21) $\dfrac{\sqrt{3}}{\sqrt{2^3\times3}-\sqrt{2^4\times3}} = \dfrac{\sqrt{3}}{2\sqrt{6}-4\sqrt{3}} = \dfrac{1}{2\sqrt{2}-4} = \dfrac{2\sqrt{2}+4}{(2\sqrt{2}-4)(2\sqrt{2}+4)} = \dfrac{2\sqrt{2}+4}{8-16} = \dfrac{2\sqrt{2}+4}{-8}$

$= -\dfrac{\sqrt{2}+2}{4}$

(22) $\dfrac{\sqrt{2}-1}{\sqrt{2}+1} = \dfrac{(\sqrt{2}-1)(\sqrt{2}-1)}{(\sqrt{2}+1)(\sqrt{2}-1)} = \dfrac{2-2\sqrt{2}+1}{2-1} = 3-2\sqrt{2}$

(23) $\dfrac{(2\sqrt{5}-\sqrt{2})^2}{(2\sqrt{5}+\sqrt{2})(2\sqrt{5}-\sqrt{2})} = \dfrac{20-4\sqrt{10}+2}{20-2} = \dfrac{22-4\sqrt{10}}{18} = \dfrac{11-2\sqrt{10}}{9}$

(24) $\dfrac{\sqrt{6}+2\sqrt{3}}{(2\sqrt{6}-\sqrt{3})} = \dfrac{(\sqrt{6}+2\sqrt{3})(2\sqrt{6}+\sqrt{3})}{(2\sqrt{6}-\sqrt{3})(2\sqrt{6}+\sqrt{3})} = \dfrac{12+\sqrt{18}+4\sqrt{18}+6}{24-3}$

$= \dfrac{18+5\sqrt{18}}{21} = \dfrac{18+15\sqrt{2}}{21} = \dfrac{6+5\sqrt{2}}{7}$

29 If $x=\sqrt{3}+1$, find the value of $x^2-\dfrac{1}{x^2}$.

$x^2-\dfrac{1}{x^2} = (\sqrt{3}+1)^2 - \dfrac{1}{(\sqrt{3}+1)^2}$

$= 3+2\sqrt{3}+1 - \dfrac{1}{4+2\sqrt{3}}$

$= 4+2\sqrt{3} - \dfrac{(4-2\sqrt{3})}{(4+2\sqrt{3})(4-2\sqrt{3})}$

$= 4+2\sqrt{3} - \dfrac{(4-2\sqrt{3})}{16-12}$

$= 4+2\sqrt{3} - \dfrac{(4-2\sqrt{3})}{4}$

$= 4+2\sqrt{3} - \dfrac{2-\sqrt{3}}{2}$

$= \dfrac{8+4\sqrt{3}-(2-\sqrt{3})}{2}$

$= \dfrac{6+5\sqrt{3}}{2}$

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For questions **36** to **44**, express as a single fraction with a rational denominator:

36 $\frac{1}{2\sqrt{3}-1}+\frac{3}{\sqrt{3}+1}$ **37** $\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}-\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$ **38** $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}\times\frac{2\sqrt{2}-\sqrt{3}}{2\sqrt{2}+\sqrt{3}}$

(36) $\frac{1}{2\sqrt{3}-1}+\frac{3}{\sqrt{3}+1}=\frac{2\sqrt{3}+1}{(2\sqrt{3}-1)(2\sqrt{3}+1)}+\frac{3(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}$

$=\frac{2\sqrt{3}+1}{12-1}+\frac{3(\sqrt{3}-1)}{3-1}=\frac{2\sqrt{3}+1}{11}+\frac{3(\sqrt{3}-1)}{2}$

$=\frac{2(2\sqrt{3}+1)+33(\sqrt{3}-1)}{22}=\frac{4\sqrt{3}+2+33\sqrt{3}-33}{22}=\frac{37\sqrt{3}-31}{22}$

(37) $\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}-\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}=\frac{(\sqrt{5}+\sqrt{2})(\sqrt{5}+\sqrt{2})-(\sqrt{5}-\sqrt{2})(\sqrt{5}-\sqrt{2})}{(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})}$

$=\frac{5+2\sqrt{10}+2-(5-2\sqrt{10}+2)}{5-2}$

$=\frac{7+2\sqrt{10}-7+2\sqrt{10}}{3}=\frac{4\sqrt{10}}{3}$

(38) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}\times\frac{2\sqrt{2}-\sqrt{3}}{2\sqrt{2}+\sqrt{3}}=\frac{(\sqrt{3}-\sqrt{2})(2\sqrt{2}-\sqrt{3})}{(\sqrt{3}+\sqrt{2})(2\sqrt{2}+\sqrt{3})}$

$=\frac{2\sqrt{6}-3-4+\sqrt{6}}{2\sqrt{6}+3+4+\sqrt{6}}=\frac{3\sqrt{6}-7}{3\sqrt{6}+7}$

$=\frac{(3\sqrt{6}-7)(3\sqrt{6}-7)}{(3\sqrt{6}+7)(3\sqrt{6}-7)}=\frac{54-21\sqrt{6}-21\sqrt{6}+49}{54-49}$

$=\frac{103-42\sqrt{6}}{5}$

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42 $\frac{\sqrt{3}-1}{\sqrt{3}+2}-\frac{\sqrt{5}-\sqrt{3}}{2\sqrt{5}+\sqrt{3}}$ 43 $\frac{2\sqrt{5}}{\sqrt{10}-\sqrt{15}}-\frac{3\sqrt{7}}{\sqrt{35}-\sqrt{14}}$ 44 $\frac{1}{x-1}+\frac{1}{x+1}-\frac{2}{x^2-1}$, where $x=2\sqrt{3}+1$

(42) $\frac{\sqrt{3}-1}{\sqrt{3}+2}-\frac{\sqrt{5}-\sqrt{3}}{2\sqrt{5}+\sqrt{3}}=\frac{(\sqrt{3}-1)(\sqrt{3}-2)}{3-4}-\frac{(\sqrt{5}-\sqrt{3})(2\sqrt{5}-\sqrt{3})}{(2\sqrt{5}+\sqrt{3})(2\sqrt{5}-\sqrt{3})}$

$= \frac{3+2-2\sqrt{3}-\sqrt{3}}{-1}-\frac{10-\sqrt{15}-2\sqrt{15}+3}{20-3}$

$=-5+3\sqrt{3}-\frac{13-3\sqrt{15}}{17}=\frac{-85+51\sqrt{3}-13+3\sqrt{15}}{17}$

$=\frac{-98+51\sqrt{3}+3\sqrt{15}}{17}$

(43) $\frac{2\sqrt{5}}{\sqrt{10}-\sqrt{15}}-\frac{3\sqrt{7}}{\sqrt{35}-\sqrt{14}}=\frac{2}{\sqrt{2}-\sqrt{3}}-\frac{3}{\sqrt{5}-\sqrt{2}}$

$=\frac{2(\sqrt{2}+\sqrt{3})}{2-3}-\frac{3(\sqrt{5}+\sqrt{2})}{5-2}$

$=-2(\sqrt{2}+\sqrt{3})-(\sqrt{5}+\sqrt{2})=-[\sqrt{5}+2\sqrt{3}+3\sqrt{2}]$

(44) $\frac{1}{x-1}+\frac{1}{x+1}-\frac{2}{x^2-1}=\frac{1}{2\sqrt{3}}+\frac{1}{2\sqrt{3}+2}-\frac{2}{2\sqrt{3}(2\sqrt{3}+2)}$

$=\frac{\sqrt{3}}{6}+\frac{2\sqrt{3}-2}{12-4}-\frac{2}{12+4\sqrt{3}}$

$=\frac{\sqrt{3}}{6}+\frac{\sqrt{3}-1}{4}-\frac{1}{6+2\sqrt{3}}$

$=\frac{4\sqrt{3}+6\sqrt{3}-6}{24}-\frac{6-2\sqrt{3}}{36-12}$

$=\frac{10\sqrt{3}-6}{24}-\frac{6-2\sqrt{3}}{24}$

$=\frac{10\sqrt{3}-6-6+2\sqrt{3}}{24}=\frac{12\sqrt{3}-12}{24}=\frac{\sqrt{3}-1}{2}$