SCALAR PRODUCT OF VECTORS IN THREE DIMENSIONS

1 For each of the following pairs of vectors, find **(i)** $\underset{\sim}{a} \bullet \underset{\sim}{b}$ **(ii)** the angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

(c) $\underset{\sim}{a} = 4\underset{\sim}{i} - 5\underset{\sim}{j} + 7\underset{\sim}{k}$, $\underset{\sim}{b} = 2\underset{\sim}{i} + \underset{\sim}{j} + 3\underset{\sim}{k}$ **(d)** $\underset{\sim}{a} = 6\underset{\sim}{i} - \underset{\sim}{j}$, $\underset{\sim}{b} = 2\underset{\sim}{j} - \underset{\sim}{k}$

c) i) $\vec{a} \cdot \vec{b} = 4 \times 2 + (-5) \times 1 + 7 \times 3 = 8 - 5 + 21 = 24$.

ii) $\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$

$|\vec{a}| = \sqrt{4^2 + (-5)^2 + 7^2} = \sqrt{16 + 25 + 49} = \sqrt{90} = 3\sqrt{10}$

$|\vec{b}| = \sqrt{2^2 + 1^2 + 3^2} = \sqrt{4 + 1 + 9} = \sqrt{14}$

So $\cos\theta = \dfrac{24}{3\sqrt{10} \times \sqrt{14}} = \dfrac{24}{6\sqrt{5}\sqrt{7}} = \dfrac{4}{\sqrt{35}}$ $\theta \simeq 47°28'$

d) i) $\vec{a} \cdot \vec{b} = 6 \times 0 + (-1) \times 2 + 0 \times (-1) = -2$

ii) $\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$

$|\vec{a}| = \sqrt{6^2 + (-1)^2} = \sqrt{37}$

$|\vec{b}| = \sqrt{2^2 + (-1)^2} = \sqrt{5}$

So $\cos\theta = \dfrac{-2}{\sqrt{37}\sqrt{5}} = \dfrac{-2}{\sqrt{185}}$

$\theta \simeq 98°27'$

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2 A, B, C, D are four points in space with respective coordinates $(0, 0, 0)$, $(1, 2, 3)$, $(-3, 4, 6)$, $(2, -6, -4)$.
A B C D

Find:

(a) the position vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$

(b) the magnitude of the angle between $\overrightarrow{AB}$ and $\overrightarrow{CD}$

~~**(c)** the position vectors $\overrightarrow{BC}$ and $\overrightarrow{AB}$~~

~~**(d)** the magnitude of the angle between $\overrightarrow{BC}$ and $\overrightarrow{AB}$~~

(e) the scalar projection of $\overrightarrow{AB}$ on $\overrightarrow{CD}$

(f) the scalar projection of $\overrightarrow{CD}$ on $\overrightarrow{AB}$.

a) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -\overrightarrow{OA} + \overrightarrow{OB}$

$\overrightarrow{AB} = \vec{0} + \vec{i} + 2\vec{j} + 3\vec{k} = \vec{i} + 2\vec{j} + 3\vec{k}$

$\overrightarrow{CD} = -\overrightarrow{OC} + \overrightarrow{OD} = -(3\vec{i} + 4\vec{j} + 6\vec{k}) + (2\vec{i} - 6\vec{j} - 4\vec{k})$

$\overrightarrow{CD} = 5\vec{i} - 10\vec{j} - 10\vec{k}$

b) $\cos\theta = \dfrac{\overrightarrow{AB} \cdot \overrightarrow{CD}}{|\overrightarrow{AB}|\,|\overrightarrow{CD}|}$

$|\overrightarrow{AB}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{1+4+9} = \sqrt{14}$

$|\overrightarrow{CD}| = \sqrt{(5)^2 + (-10)^2 + (-10)^2} = \sqrt{225} = 15$

$\cos\theta = \dfrac{1 \times 5 + 2 \times (-10) + 3 \times (-10)}{\sqrt{14} \cdot 15} = \dfrac{-45}{15\sqrt{14}} = \dfrac{-3}{\sqrt{14}}$ $\theta \simeq 143^\circ 18'$

e) The scalar projection of $\overrightarrow{AB}$ onto $\overrightarrow{CD}$ is $\dfrac{\overrightarrow{AB} \cdot \overrightarrow{CD}}{|\overrightarrow{CD}|}$

So it's $\dfrac{-45}{15} = -3$

f) The scalar projection of $\overrightarrow{CD}$ on $\overrightarrow{AB}$ is $\dfrac{\overrightarrow{CD} \cdot \overrightarrow{AB}}{|\overrightarrow{AB}|}$

So it's $\dfrac{-45}{\sqrt{14}} = -\dfrac{45\sqrt{14}}{14}$

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5 If $\underset{\sim}{a} = \underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}$, $\underset{\sim}{b} = 2\underset{\sim}{i} - 3\underset{\sim}{j} + 4\underset{\sim}{k}$ and $\underset{\sim}{c} = -2\underset{\sim}{i} - \underset{\sim}{j} + 3\underset{\sim}{k}$, find:

(a) $(\underset{\sim}{c} - \underset{\sim}{a}) \bullet \underset{\sim}{b}$ **(b)** $(\underset{\sim}{a} + \underset{\sim}{b}) \bullet \underset{\sim}{c}$ **(c)** the scalar projection of $\underset{\sim}{a}$ onto $\underset{\sim}{b}$.

a) $(\vec{c} - \vec{a}) . \vec{b} = [-2\vec{i} - \vec{j} + 3\vec{k} - (\vec{i} + \vec{j} + \vec{k})] . (2\vec{i} - 3\vec{j} + 4\vec{k})$

$= [-3\vec{i} - 2\vec{j} + 2\vec{k}] . (2\vec{i} - 3\vec{j} + 4\vec{k})$

$= -6 + 6 + 8 = 8$

b) $(\vec{a} + \vec{b}) . \vec{c} = [\vec{i} + \vec{j} + \vec{k} + 2\vec{i} - 3\vec{j} + 4\vec{k}] . \vec{c}$

$= [3\vec{i} - 2\vec{j} + 5\vec{k}] . (-2\vec{i} - \vec{j} + 3\vec{k})$

$= -6 + 2 + 15 = 11$

c) The scalar projection of $\vec{a}$ onto $\vec{b}$ is $\dfrac{\vec{a} . \vec{b}}{|\vec{b}|}$

So it's $\dfrac{1 \times 2 + 1 \times (-3) + 1 \times 4}{\sqrt{2^2 + (-3)^2 + 4^2}} = \dfrac{2 - 3 + 4}{\sqrt{4 + 9 + 16}} = \dfrac{3}{\sqrt{29}}$

or in a rationalised form $\dfrac{3\sqrt{29}}{29}$

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6 If $\underset{\sim}{a} = 3\underset{\sim}{i} + 6\underset{\sim}{j} + 2\underset{\sim}{k}$ and $\underset{\sim}{b} = -6\underset{\sim}{i} + 2\underset{\sim}{j} + 3\underset{\sim}{k}$, find:

(a) a unit vector parallel to $2\underset{\sim}{a} + \underset{\sim}{b}$

(b) a unit vector perpendicular to both $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

a) $2\vec{a} + \vec{b} = 2(3\vec{i} + 6\vec{j} + 2\vec{k}) - 6\vec{i} + 2\vec{j} + 3\vec{k} = 14\vec{j} + 7\vec{k}$

$|2\vec{a} + \vec{b}| = \sqrt{14^2 + 7^2} = \sqrt{245} = 7\sqrt{5}$

So it's $(14\vec{j} + 7\vec{k})/7\sqrt{5} = \frac{2}{\sqrt{5}}\vec{i} + \frac{1}{\sqrt{5}}\vec{j}$

b) we must have $\vec{a}.\vec{u} = 0$ so $3x + 6y + 2z = 0$ ①

and $\vec{b}.\vec{u} = 0$ so $-6x + 2y + 3z = 0$ ②

$2 \times$ ① + ② $\Rightarrow$ $12y + 2y + 7z = 0$ so $14y + 7z = 0$ $z = -2y$.

① becomes $3x + 6y - 4y = 0$ so $3x + 2y = 0$ $y = -\frac{3}{2}x$ and so $z = 3x$

A vector perpendicular to $\vec{a}$ and $\vec{b}$ is $\vec{i} - \frac{3}{2}\vec{j} + 3\vec{k}$

$|\vec{i} - \frac{3}{2}\vec{j} + 3\vec{k}| = \sqrt{1 + (\frac{3}{2})^2 + 9} = \frac{7}{2}$

So a unit vector perpendicular to $\vec{a}$ and $\vec{b}$ is $\frac{2}{7}(\vec{i} - \frac{3}{2}\vec{j} + 3\vec{k})$

7 If $|\underset{\sim}{a}| = |\underset{\sim}{b}|$, simplify the following expressions.

(a) $(\underset{\sim}{a} + \underset{\sim}{b}) \bullet (\underset{\sim}{a} - \underset{\sim}{b})$ **(b)** $(\underset{\sim}{a} + 2\underset{\sim}{b}) \bullet (2\underset{\sim}{a} - \underset{\sim}{b})$

a) $(\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = (\vec{a} + \vec{b}).\vec{a} + (\vec{a} + \vec{b}).(-\vec{b})$

$= |\vec{a}|^2 + \vec{b}.\vec{a} - \vec{a}.\vec{b} - |\vec{b}|^2 = |\vec{a}|^2 - |\vec{b}|^2 = 0$

b) $(\vec{a} + 2\vec{b}).(2\vec{a} - \vec{b}) = (\vec{a} + 2\vec{b}).2\vec{a} + (\vec{a} + 2\vec{b}).(-\vec{b})$

$= 2|\vec{a}|^2 + 4\vec{b}.\vec{a} - \vec{a}.\vec{b} - 2|\vec{b}|^2$

$= 2|\vec{a}|^2 + 3\vec{a}.\vec{b} - 2|\vec{b}|^2$

$= 3\vec{a}.\vec{b}$

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11 If $\underset{\sim}{u} = \underset{\sim}{i} + 2\underset{\sim}{j} - 2\underset{\sim}{k}$ and $\underset{\sim}{v} = 2\underset{\sim}{i} + 3\underset{\sim}{j} - 6\underset{\sim}{k}$, find:

(a) $\hat{\underset{\sim}{u}}$ **(b)** $\hat{\underset{\sim}{v}}$

(c) a unit vector in the direction $2\underset{\sim}{u} - \underset{\sim}{v}$

(d) **(i)** the vector projection of $\underset{\sim}{u}$ parallel to $\underset{\sim}{v}$

(ii) the vector projections of $\underset{\sim}{u}$ perpendicular to $\underset{\sim}{v}$.

a) $|\vec{u}| = \sqrt{1^2 + 2^2 + 2^2} = \sqrt{9} = 3$ so $\hat{\underset{\sim}{u}} = \dfrac{\vec{i} + 2\vec{j} - 2\vec{k}}{3}$

b) $|\vec{v}| = \sqrt{2^2 + 3^2 + 6^2} = \sqrt{4 + 9 + 36} = \sqrt{49} = 7$

so $\hat{\underset{\sim}{v}} = \dfrac{2\vec{i} + 3\vec{j} - 6\vec{k}}{7}$

c) $2\vec{u} - \vec{v} = 2(\vec{i} + 2\vec{j} - 2\vec{k}) - (2\vec{i} + 3\vec{j} - 6\vec{k})$

$= \vec{j} + 2\vec{k}$

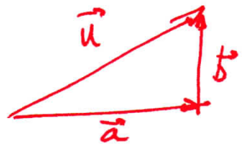
$|2\vec{u} - \vec{v}| = \sqrt{1^2 + 2^2} = \sqrt{5}$ so it'd be $\dfrac{\vec{j} + 2\vec{k}}{\sqrt{5}}$

d) The vector projection of $\vec{u}$ onto $\vec{v}$ is $|\vec{u}| \times \cos\theta \times \dfrac{\vec{v}}{|\vec{v}|}$

where $\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}$ so it's $\dfrac{\vec{u}\cdot\vec{v}}{|\vec{v}|^2} \cdot \vec{v}$

$\dfrac{\vec{u}\cdot\vec{v}}{|\vec{v}|} \times \vec{v} = \dfrac{1\times 2 + 2\times 3 - 2\times(-6)}{7^2} \times (2\vec{i} + 3\vec{j} - 6\vec{k}) = \dfrac{20}{49}(2\vec{i} + 3\vec{j} - 6\vec{k})$

e) This is $\vec{b} = -\vec{a} + \vec{u}$



So $\vec{b} = -\dfrac{20}{49}(2\vec{i} + 3\vec{j} - 6\vec{k}) + \vec{i} + 2\vec{j} - 2\vec{k}$

$\vec{b} = \dfrac{9}{49}\vec{i} + \dfrac{38}{49}\vec{j} + \dfrac{22}{49}\vec{k}$

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13 Find the value of p for which $\underset{\sim}{i} - 2p\underset{\sim}{j} + 3\underset{\sim}{k}$ and $p\underset{\sim}{i} - 4\underset{\sim}{j} + 3\underset{\sim}{k}$ are perpendicular.

These 2 vectors are perpendicular when the dot product is zero.

i.e. $1 \times p - 2p \times (-4) + 3 \times 3 = 0$

$9p + 9 = 0$ so $p = -1$

15 If $\underset{\sim}{a} = (6, -2, 6)$ and $\underset{\sim}{b} = (-6, -2, 1)$, find:

(a) $|\underset{\sim}{a}|$

(b) the scalar projection of $\underset{\sim}{b}$ on to $\underset{\sim}{a}$

(c) $|\underset{\sim}{a} - \underset{\sim}{b}|$

(d) the magnitude of the projection of $\underset{\sim}{b}$ on to $\underset{\sim}{a} - \underset{\sim}{b}$.

a) $|\vec{a}| = \sqrt{6^2 + 2^2 + 6^2} = \sqrt{76} = 2\sqrt{19}$

b) $\dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}|} = \dfrac{-36 + 4 + 6}{2\sqrt{19}} = \dfrac{-26}{2\sqrt{19}} = \dfrac{-13}{\sqrt{19}} = -\dfrac{13\sqrt{19}}{19}$

c) $|\vec{a} - \vec{b}| = |6\vec{i} - 2\vec{j} + 6\vec{k} - (-6\vec{i} - 2\vec{j} + \vec{k})| = |12\vec{i} + 5\vec{k}| = \sqrt{12^2 + 5^2} = 13$

d) This is $\left|\dfrac{\vec{b} \cdot (\vec{a} - \vec{b})}{|\vec{a} - \vec{b}|}\right| = \left|\dfrac{(-6\vec{i} - 2\vec{j} + \vec{k}) \cdot (12\vec{i} + 5\vec{k})}{13}\right|$

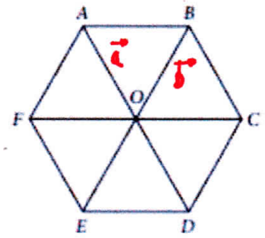
$= \left|\dfrac{-72 + 5}{13}\right| = \left|\dfrac{-67}{13}\right| = 5\dfrac{2}{13}$

17 $ABCDEF$ is a regular hexagon with centre O. If $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OB} = \underset{\sim}{b}$, express each of the following in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$:

(a) $\overrightarrow{AB}$ (b) $\overrightarrow{BC}$

(c) $\overrightarrow{CD}$ (d) $\overrightarrow{BD}$

(e) $\overrightarrow{FC}$ (f) Prove that $\overrightarrow{BD}$ and $\overrightarrow{FC}$ are perpendicular.



a) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -\overrightarrow{OA} + \overrightarrow{OB} = -\vec{a} + \vec{b}$

b) $\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC} = -\overrightarrow{OB} + \overrightarrow{OC} = -\vec{b} + \overrightarrow{AB} = -\vec{b} + (-\vec{a} + \vec{b}) = -\vec{a}$

c) $\overrightarrow{CD} = -\vec{b}$

d) $\overrightarrow{BD} = \overrightarrow{BC} + \overrightarrow{CD} = -\vec{a} - \vec{b}$

e) $\overrightarrow{FC} = 2 \times \overrightarrow{OC} = 2\overrightarrow{AB} = 2(-\vec{a} + \vec{b})$

f) $\overrightarrow{BD} \cdot \overrightarrow{FC} = (-\vec{a} - \vec{b}) \cdot (2(-\vec{a} + \vec{b}))$

$= (-\vec{a} - \vec{b}) \cdot (-2\vec{a}) + (-\vec{a} - \vec{b}) \cdot (2\vec{b})$

$= 2|\vec{a}|^2 + 2\vec{b} \cdot \vec{a} - 2\vec{a} \cdot \vec{b} - 2|\vec{b}|^2$

$= 2\left[|\vec{a}|^2 - |\vec{b}|^2\right]$

But $|\vec{a}| = |\vec{b}|$ $\therefore$ $\overrightarrow{BD} \cdot \overrightarrow{FC} = 0$

$\therefore$ $\overrightarrow{BD}$ and $\overrightarrow{FC}$ are perpendicular.

SCALAR PRODUCT OF VECTORS IN THREE DIMENSIONS

18 Given $\underset{\sim}{a} = 2\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$ and $\underset{\sim}{b} = \underset{\sim}{i} + 2\underset{\sim}{j} - 2\underset{\sim}{k}$, find two vectors $\underset{\sim}{c}$ and $\underset{\sim}{d}$ such that $\underset{\sim}{a} = \underset{\sim}{c} + \underset{\sim}{d}$, $\underset{\sim}{c}$ is parallel to $\underset{\sim}{b}$, $\underset{\sim}{d}$ is perpendicular to $\underset{\sim}{b}$.

$\vec{c}$ is $/\!/$ to $\vec{b}$, $\therefore \exists k \in \mathbb{R}$ such that $\vec{c} = k\vec{b} = k(\vec{i} + 2\vec{j} - 2\vec{k})$

$\vec{d}$ is $\perp$ to $\vec{b}$, $\therefore \vec{d}\cdot\vec{b} = 0$.

Say $\vec{d} = x\vec{i} + y\vec{j} + z\vec{k}$, $\therefore x + 2y - 2z = 0$

Further $\vec{a} = \vec{c} + \vec{d}$

So $2\vec{i} - \vec{j} + 2k = k\vec{i} + 2k\vec{j} - 2k\vec{k} + x\vec{i} + y\vec{j} + z\vec{k}$

So $2\vec{i} - \vec{j} + 2\vec{k} = (k+x)\vec{i} + (2k+y)\vec{j} + (-2k+z)\vec{k}$

$$\therefore \begin{cases} k + x = 2 \\ 2k + y = -1 \\ -2k + z = 2 \end{cases} \quad \text{OR} \quad \begin{cases} x = 2 - k \\ y = -1 - 2k \\ z = 2 + 2k \end{cases}$$

So $(2-k) + 2(-1-2k) - 2(2+2k) = 0$

$(-1-4-4)k + 2 - 2 - 4 = 0$

$-9k = 4$ $\qquad k = -4/9$

$$\therefore \begin{cases} x = 2 - (-4/9) = \frac{22}{9} \\ y = -1 - 2(-4/9) = -1 + \frac{8}{9} = -\frac{1}{9} \\ z = 2 + 2 \times \left(\frac{-4}{9}\right) = 2 - \frac{8}{9} = \frac{10}{9} \end{cases}$$

$\therefore \vec{c} = -\frac{4}{9}(\vec{i} + 2\vec{j} - 2\vec{k})$ and $\vec{d} = \frac{22}{9}\vec{i} - \frac{1}{9}\vec{j} + \frac{10}{9}\vec{k}$

Check ~~then~~ $\vec{c} + \vec{d} = \frac{18}{9}\vec{i} + \left(\frac{-1}{9} - \frac{8}{9}\right)\vec{j} + \left(\frac{10}{9} + \frac{8}{9}\right)\vec{k} = 2\vec{i} - \vec{j} + 2\vec{k} = \vec{a}$ indeed

22 $ABCD$ is a rectangle with vector $\overrightarrow{AB} = 3\underset{\sim}{i}$ and vector $\overrightarrow{AD} = 2\underset{\sim}{j}$.

(a) Express the diagonal vectors $\overrightarrow{AC}$ and $\overrightarrow{DB}$ in terms of $\underset{\sim}{i}$ and $\underset{\sim}{j}$.

(b) Calculate, to the nearest degree, the angle between the diagonals.

a) $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AB} + \overrightarrow{AD}$

$\overrightarrow{AC} = 3\vec{i} + 2\vec{j}$

$\overrightarrow{DB} = \overrightarrow{DA} + \overrightarrow{AB} = -\overrightarrow{AD} + \overrightarrow{AB} = -2\vec{j} + 3\vec{i}$

b) $\cos\theta = \dfrac{\overrightarrow{AC} \cdot \overrightarrow{DB}}{|\overrightarrow{AC}||\overrightarrow{DB}|} = \dfrac{(3\vec{i} + 2\vec{j}).(3\vec{i} - 2\vec{j})}{\sqrt{3^2 + 2^2}\ \sqrt{(-2)^2 + 3^2}}$

$\cos\theta = \dfrac{9 - 4}{(9 + 4)} = \dfrac{5}{13}$

$\therefore\ \theta = \cos^{-1}\left(\dfrac{5}{13}\right) = 67^\circ 23'$

so 67° to the nearest degree

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23 If $\underset{\sim}{u}$ and $\underset{\sim}{v}$ are vectors defined by $\underset{\sim}{u} = \underset{\sim}{i} + \underset{\sim}{j} + \sqrt{2}\underset{\sim}{k}$ and $\underset{\sim}{v} = \underset{\sim}{i} - \underset{\sim}{j} + \sqrt{2}\underset{\sim}{k}$, find:

(a) a unit vector parallel to $\underset{\sim}{u}$

(b) the angle between $\underset{\sim}{u}$ and $\underset{\sim}{v}$

(c) the vector projection of $\underset{\sim}{v}$ in the direction of $\underset{\sim}{u}$.

a) $\hat{\underset{\sim}{u}} = \dfrac{\vec{u}}{|\vec{u}|} = \dfrac{\vec{i} + \vec{j} + \sqrt{2}\vec{k}}{\sqrt{1^2 + 1^2 + (\sqrt{2})^2}} = \dfrac{1}{\sqrt{4}}(\vec{i} + \vec{j} + \sqrt{2}\vec{k})$

$\hat{\underset{\sim}{u}} = \dfrac{1}{2}(\vec{i} + \vec{j} + \sqrt{2}\vec{k})$

b) $\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|} = \dfrac{(\vec{i} + \vec{j} + \sqrt{2}\vec{k}).(\vec{i} - \vec{j} + \sqrt{2}\vec{k})}{2 \times 2}$

$\cos\theta = \dfrac{1 - 1 + 2}{4} = \dfrac{1}{2}$ so $\theta = \dfrac{\pi}{3}$ or 60°

c) This is $|\vec{v}| \cos 60 \times \hat{\underset{\sim}{u}}$.

$|\vec{v}| \cos 60 \times \hat{\underset{\sim}{u}} = 2 \times \dfrac{1}{2} \times \dfrac{1}{2}(\vec{i} + \vec{j} + \sqrt{2}\vec{k})$

$= \dfrac{1}{2}(\vec{i} + \vec{j} + \sqrt{2}\vec{k})$