# TRIGONOMETRIC EQUATIONS INVOLVING ANGLE FORMULAE

## Example 23

Solve the equation $\sin 2x = 3\cos x$, $0 \le x \le 2\pi$.

### Solution

As $\sin 2x = 2\sin x\cos x$: $2\sin x\cos x = 3\cos x$

$$\cos x(2\sin x - 3) = 0$$

$$\therefore \cos x = 0 \text{ or } \sin x = 1.5$$

Because $|\sin x| \le 1$, the only solution is $\cos x = 0$.

$$\therefore x = \frac{\pi}{2}, \frac{3\pi}{2}$$

## Example 24

Solve the equation $\cos 2x\cos\alpha - \sin 2x\sin\alpha = -0.5$, $0 \le x \le 2\pi$, where $\alpha = \frac{\pi}{6}$.

### Solution

Use the expansion of $\cos(A + B)$ to simplify the LHS:

$\cos 2x\cos\alpha - \sin 2x\sin\alpha = \cos(2x + \alpha)$

Hence, as $\alpha = \frac{\pi}{6}$: $\cos\left(2x + \frac{\pi}{6}\right) = -0.5$

$$2x + \frac{\pi}{6} = \frac{2\pi}{3}, \frac{4\pi}{3}, 2\pi + \frac{2\pi}{3}, 2\pi + \frac{4\pi}{3}$$

$$2x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{5\pi}{2}, \frac{19\pi}{6}$$

$$x = \frac{\pi}{4}, \frac{7\pi}{12}, \frac{5\pi}{4}, \frac{19\pi}{12}$$

## Example 25

Solve for $0 \le \theta \le \pi$, the equation $\sin 2\theta\cos\theta = \sin 3\theta\cos 2\theta$.

### Solution

$\sin 2\theta\cos\theta = \sin 3\theta\cos 2\theta$.

Convert each product to a sum: $\frac{1}{2}(\sin(2\theta + \theta) + \sin(2\theta - \theta)) = \frac{1}{2}(\sin(3\theta + 2\theta) + \sin(3\theta - 2\theta))$

$$\sin 3\theta + \sin\theta = \sin 5\theta + \sin\theta$$

Simplify: $\sin 3\theta = \sin 5\theta$

Rewrite: $\sin 5\theta = \sin 3\theta$

Solve: $5\theta = 3\theta, \pi - 3\theta, 2\pi + 3\theta, 3\pi - 3\theta, 4\pi + 3\theta, 5\pi - 3\theta, 6\pi + 3\theta, 7\pi - 3\theta$.

$0 \le \theta \le \pi$: $2\theta = 0, 2\pi, 4\pi, \ldots$

$$\theta = 0, \pi$$

and $8\theta = \pi, 3\pi, 5\pi, 7\pi$.

$$\theta = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}.$$

The complete solution is: $\theta = 0, \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \pi$.

## Example 26

Solve for $0 \leq \theta \leq \pi$, the equation $\cos 3\theta + \cos \theta = \cos 2\theta$.

### Solution

$\cos 3\theta + \cos \theta = \cos 2\theta$

Convert the LHS to a product: $2\cos\left(\frac{3\theta+\theta}{2}\right)\cos\left(\frac{3\theta-\theta}{2}\right) = \cos 2\theta$

$$2\cos 2\theta \cos \theta = \cos 2\theta$$

Rearrange: $\cos 2\theta\,(2\cos \theta - 1) = 0$

Solve: $\cos 2\theta = 0$ or $\cos \theta = \frac{1}{2}$

$2\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ or $\theta = \frac{\pi}{3}$.

The complete solution is: $\theta = \frac{\pi}{4}, \frac{\pi}{3}, \frac{3\pi}{4}$.