POLYNOMIALS

1 For the polynomial $P(x) = 3x^4 + 2x^3 + 7$, which statement is correct?

A degree = 3 **B** leading term = 3 **C** leading coefficient = 3 **D** constant term = 3

A: FALSE it's 4
B: False it's $3x^4$
C: true
D: False it's 7

2 Express the polynomial $P(x) = x^2 - x^3 + 6x$ in standard form. Then write:

(a) its degree **(b)** the constant term **(c)** the coefficient of x^2 **(d)** the leading term
(e) the greatest number of real zeros possible. **(f)** Hence solve the equation $P(x) = 0$.

$P(x) = -x^3 + x^2 + 6x$

a) degree 3 b) 0 c) 1 d) $-x^3$

e) it has at most 3 zeros as it's of degree 3

f) $P(x) = -x^3 + x^2 + 6x = x(-x^2 + x + 6)$

$\Delta = 1 - 4 \times (-1) \times 6 = 25 = 5^2$ so two roots

$x_1 = \dfrac{-1-5}{-2} = 3$ and $x_2 = \dfrac{-1+5}{-2} = -2$

$P(x) = x(x-3)(x+2)(-1)$ so there are 3 solutions to $P(x) = 0$ which are $x = 0$, $x = -2$ and $x = 3$

3 Write the following polynomials in standard form and then state:

(i) the degree **(ii)** the constant term **(iii)** the coefficient of x^2
(iv) whether or not it is monic **(v)** the greatest number of real zeros possible.

(a) $x^2 + 5x^3 + 7 - 6x$ **(b)** $27 - x^3$ **(c)** $ax^3 + bx + cx^2 - d$

a) $P(x) = 5x^3 + x^2 - 6x + 7$ degree 3, 7 is the constant term, 1 is coefficient of x^2 not monic, 3 zeros at most.

b) $P(x) = 27 - x^3$ degree 3, 27 is constant term, 0 is coefficient of x^2 not monic, 3 zeros at most

c) $P(x) = ax^3 + cx^2 + bx - d$ degree 3, $-d$ is constant term

c is coefficient of x^2, not monic except if $a = 1$,

3 zeros at most.

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4 State whether each expression is a polynomial or not. If it is not a polynomial, explain why.

(a) x^2-6x+3 (b) $x+4$ (c) $\sqrt{3}x-4$ (d) x^9+1 (e) $4-\frac{1}{x}$

(f) $\frac{6x+2}{3}$ (g) $x^2+3x^{\frac{1}{2}}-4x^{-1}$ (h) $\frac{3x+2}{3x-1}$ (i) 2^x+3x-5

a) yes. c) yes d) yes e) NO as it's degree is not a natural number

b) yes

f) yes as it can be rewritten $2x+\frac{2}{3}$

g) no as it has a term in $x^{1/2}$ and another in x^{-1}

h) no

i) no as it has a term 2^x

5 If $A(x)=x^2-5x+1$ and $B(x)=3x^4-2x^2+5x+3$, then $B(x)-A(x)=\ldots$

A $-3x^4+3x^2-10x-2$ B $3x^4-3x^2+2$ C $3x^4-3x^2-10x+2$ $\boxed{\text{D}}$ $3x^4-3x^2+10x+2$

$B(x)-A(x)=3x^4-2x^2+5x+3-(x^2-5x+1)$

$=3x^4-3x^2+10x+2$

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6 If $A(x) = 3x^2 - 2x + 1$, $B(x) = 5x - 2$, $C(x) = 2x^4 - 5x^2 + 3x + 4$ and $D(x) = 2x^5 - 4x^2 - 3$, simplify:

(a) $A(x) + C(x)$ **(b)** $B(x) \times D(x)$ **(c)** $D(x) - C(x)$
(d) $A(x) \times B(x)$ **(e)** $A(x) - 3C(x) + 2B(x)$

a) $A(x) + C(x) = 3x^2 - 2x + 1 + 2x^4 - 5x^2 + 3x + 4$

$A(x) + C(x) = 2x^4 - 2x^2 + x + 5$

b) $B(x) \times D(x) = (5x-2)(2x^5 - 4x^2 - 3)$

$= 10x^6 - 20x^3 - 15x - 4x^5 + 8x^2 + 6$

$= 10x^6 - 4x^5 - 20x^3 + 8x^2 - 15x + 6$

c) $D(x) - C(x) = 2x^5 - 4x^2 - 3 - [2x^4 - 5x^2 + 3x + 4]$

$= 2x^5 - 4x^2 - 3 - 2x^4 + 5x^2 - 3x - 4$

$= 2x^5 - 2x^4 + x^2 - 3x - 7$

d) $A(x) \times B(x) = (3x^2 - 2x + 1)(5x - 2)$

$= 15x^3 - 6x^2 - 10x^2 + 4x + 5x - 2$

$= 15x^3 - 16x^2 + 9x - 2$

e) $A(x) - 3C(x) + 2B(x) = 3x^2 - 2x + 1 - 3(2x^4 - 5x^2 + 3x + 4)$
$+ 2(5x - 2)$

$= 3x^2 - 2x + 1 - 6x^4 + 15x^2 - 9x - 12 + 10x - 4$

$= -6x^4 + 18x^2 - x - 15$

7 If $E(x) = x^2 - 3$, $F(x) = 3x + 2$, $G(x) = x^2 + 2x + 1$ and $H(x) = x^2 - 3x + 2$, find the polynomial for:

(a) $E(x) \times F(x)$ **(b)** $F(x) \times G(x)$ **(c)** $3G(x) - 4H(x)$
(d) $(x - 3)G(x)$ **(e)** $[F(x)]^2$ **(f)** $E(x) \times G(x) + F(x) \times H(x)$

a) $E(x)\,F(x) = (x^2-3)(3x+2) = 3x^3 + 2x^2 - 9x - 6$

b) $F(x)\,G(x) = (3x+2)(x^2+2x+1)$

$= 3x^3 + 6x^2 + 3x + 2x^2 + 4x + 2$

$= 3x^3 + 8x^2 + 7x + 2$

c) $3\,G(x) - 4\,H(x) = 3[x^2+2x+1] - 4[x^2-3x+2]$

$= 3x^2 + 6x + 3 - 4x^2 + 12x - 8$

$= -x^2 + 18x - 5$

d) $(x-3)\,G(x) = (x-3)(x^2+2x+1)$

$= x^3 + 2x^2 + x - 3x^2 - 6x - 3$

$= x^3 - x^2 - 5x - 3$

e) $[F(x)]^2 = [3x+2]^2 = 9x^2 + 12x + 4$

f) $E(x)\,G(x) + F(x)\,H(x) = (x^2-3)(x^2+2x+1) + (3x+2)(x^2-3x+2)$

$= x^4 + 2x^3 + \cancel{x^2} - \cancel{3x^2} - \cancel{6x} - 3 + 3x^3 - 9x^2 + \cancel{6x}$
$+ \cancel{2x^2} - 6x + 4$

$= x^4 + 5x^3 - 9x^2 - 6x + 1$