CURVES AND REGIONS ON THE ARGAND DIAGRAM

1 If $z = x + iy$, the Cartesian equation $x - y = 0$ represents:

A $\arg z = \frac{\pi}{4}$ $\boxed{\textbf{B}}$ $|z+2i| = |z+2|$ **C** $\arg z = -\frac{3\pi}{4}$ **D** $|z+2i| = |z-2|$

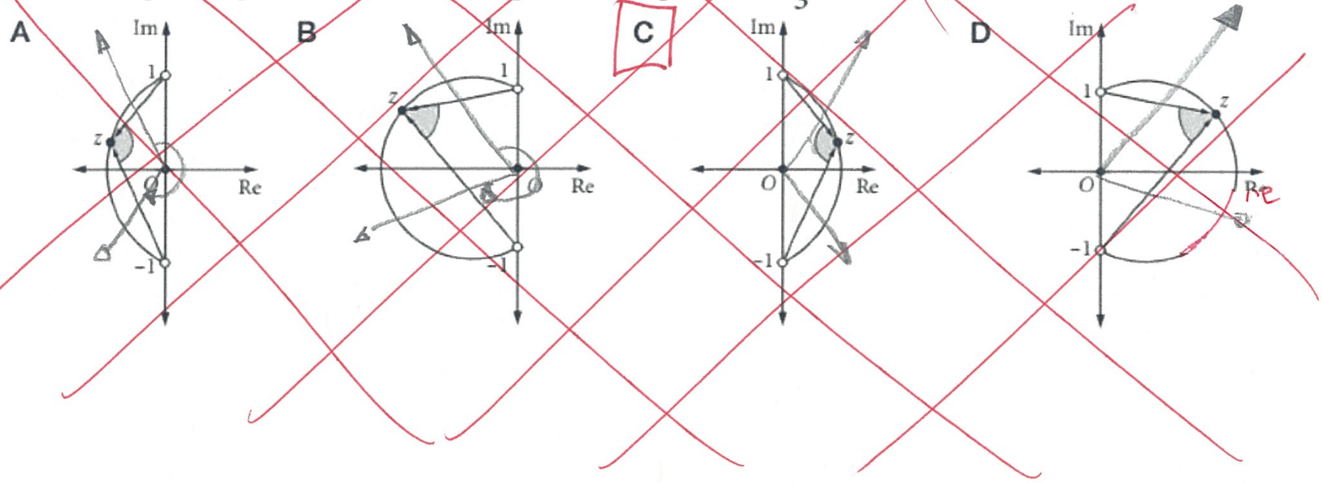
incomplete   incomplete

$|z+2i| = \sqrt{x^2+(y+2)^2} = \sqrt{x^2+y^2+4y+4}$ } equal as $y = x$

whereas $|z+2| = \sqrt{(x+2)^2+y^2} = \sqrt{x^2+y^2+4x+4}$

$|z-2| = \sqrt{x^2-4x+4+y^2}$ NO

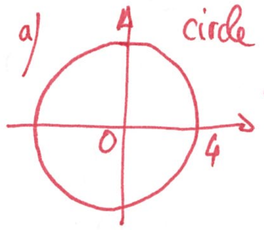
2 Which diagram represents z such that $\arg(z+i) - \arg(z-i) = \frac{2\pi}{3}$?



3 On Argand diagrams, show the curves or regions described by the following.

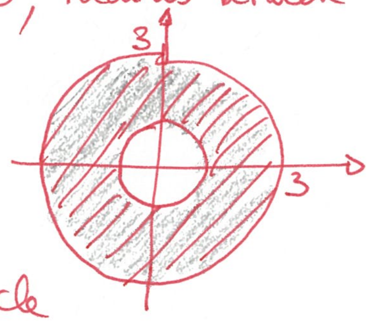
(a) $|z| = 4$ **(b)** $|z| \le 2$ **(c)** $1 \le |z| \le 3$ **(d)** $|z - (1+\sqrt{3}i)| = 2$ **(e)** $|z - 2 + 2i| = 3$

a) circle centered on O of radius 4



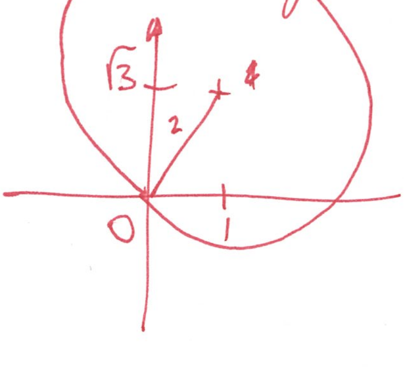
b) ~~disc~~ centered on O of radius 2, excluding this circle.

c) Washer centered on O, radius between 1 and 3 included



d) ~~disc~~ circle centered on $(1+\sqrt{3}i)$, of radius 2

Note that the distance from the centre of the circle to the origin O is $\sqrt{1^2+3} = \sqrt{4} = 2$ so same as the radius, $\therefore$ O belongs to this circle



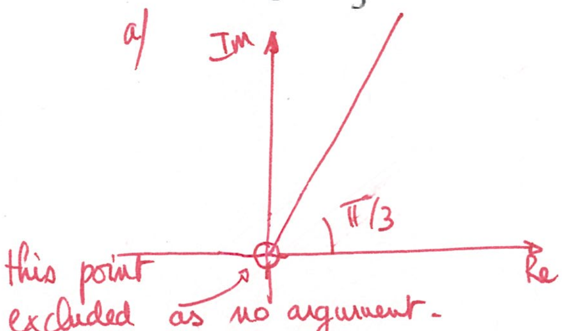
e) $\Leftrightarrow |z-(2-2i)| = 3$

circle centered on $2-2i$, of radius 3

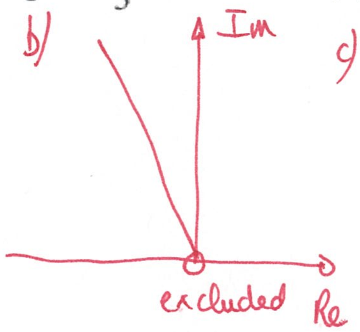
4 On Argand diagrams, show the curves or regions described by the following.

(a) $\arg z = \frac{\pi}{3}$ **(b)** $\arg z = \frac{2\pi}{3}$ **(c)** $-\frac{\pi}{3} \le \arg z \le \frac{2\pi}{3}$

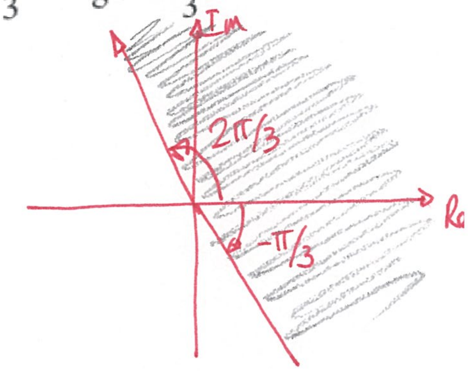
a)



b)



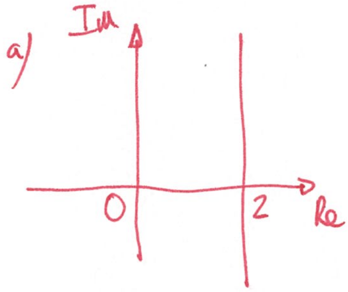
c)



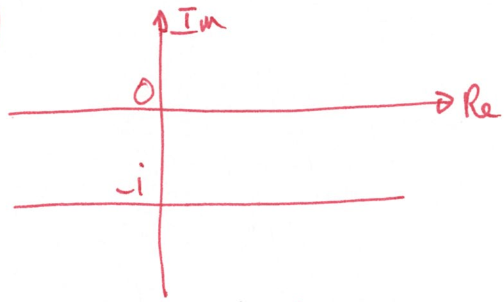
5 Show the following on the complex plane.

(a) $\text{Re}(z) = 2$ **(b)** $\text{Im}(z) = -1$ **(c)** $\text{Re}(z) + \text{Im}(z) = 1$ **(d)** $\text{Re}(z) < \text{Im}(z)$ **(e)** $z + \bar{z} = 6$

a)



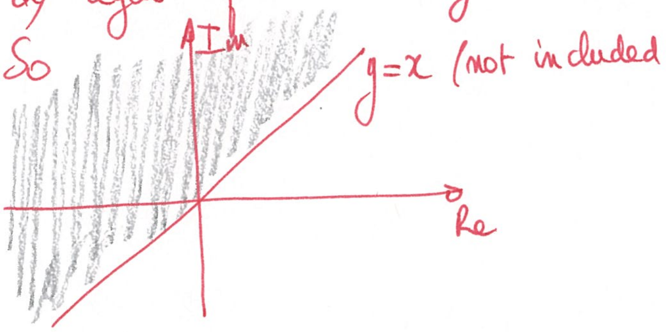
b)



c) if $z = x + iy$ then $\text{Re}(z) + \text{Im}(z) = x + y = 1$ so $y = -x + 1$ which is the line of gradient (-1), y-intercept 1

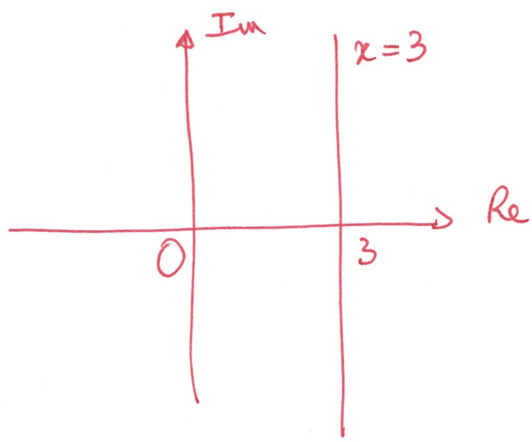
d) again if $z = x + iy$ $\text{Re}(z) < \text{Im}(z) \Leftrightarrow x < y$.

So



e) if $z = x + iy$ then $z + \bar{z} = x + iy + x - iy = 2x$

So $z + \bar{z} = 6 \Leftrightarrow 2x = 6$ or $x = 3$

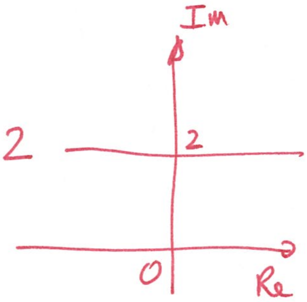


CURVES AND REGIONS ON THE ARGAND DIAGRAM

(f) $z-\bar{z}=4i$ **(g)** $2|z|=z+\bar{z}+4$ **(h)** $\left|z^2-(\bar{z})^2\right|\geq 16$ **(i)** $|z+2-4i|=2|z-4-i|$

f) if $z=x+iy$ $\quad z-\bar{z}=x+iy-(x-iy)=2iy$.

So $z-\bar{z}=4i \iff 2iy=4i$ or $y=2$



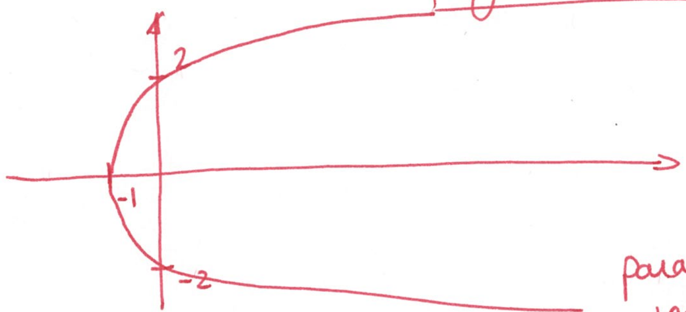
g) if $z=x+iy$

$2|z|=z+\bar{z}+4 \iff 2\sqrt{x^2+y^2}=2x+4$

or $4(x^2+y^2)=(2x+4)^2=4x^2+16x+16$

or $4y^2=16x+16$ or $\boxed{y^2=4x+4}$

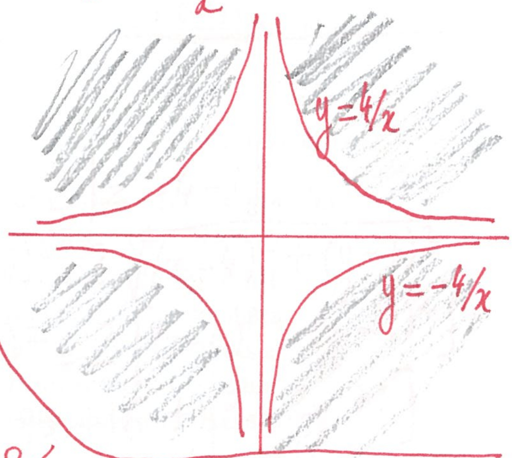
$y^2=4(x+1)$



parabola of vertex $(-1,0)$.

h) $\iff |(z-\bar{z})(z+\bar{z})|\geqslant 16 \iff |2iy\times 2x|\geqslant 16 \iff |ixy|\geqslant 4$

or $|yx|\geqslant 4$ So we draw $y=\frac{4}{x}$ and $y=-\frac{4}{x}$



i) $\iff |z-(-2+4i)|=2\,|z-(4+i)|$

$\iff \sqrt{(x+2)^2+(y-4)^2}=2\sqrt{(x-4)^2+(y-1)^2}$

$\iff (x+2)^2+(y-4)^2=4\left[(x-4)^2+(y-1)^2\right]$

$\iff x^2+4x+4+y^2-8y+16=$
$4\left[x^2-8x+16+y^2-2y+1\right]$

$\iff x^2+4x+y^2-\cancel{8y}+20=4x^2-32x+4y^2-\cancel{8y}+68$

$\iff 0=3x^2+3y^2-36x+48$

$\iff x^2+y^2-12x+16=0$

$\iff (x-6)^2+y^2-36+16=0$

$\iff (x-6)^2+y^2=20=(2\sqrt{5})^2$

So circle centered on $(6,0)$, of radius $2\sqrt{5}$

6 On Argand diagrams, show:

(a) the region where $|z-1| \leq 1$ and $\mathrm{Im}(z) \geq 0$ are both true

(b) the intersection of $2 \leq |z| \leq 3$ and $-\frac{\pi}{2} \leq \arg z \leq \frac{\pi}{4}$

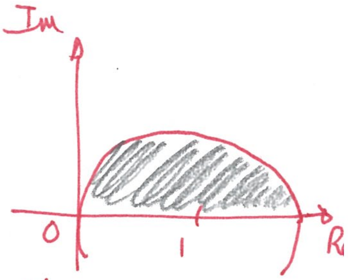
(c) the intersection of $-\frac{\pi}{3} \leq \arg z \leq \frac{\pi}{3}$ and $\mathrm{Re}(z) < 2$

(d) the intersection of $|z| \leq 3$ and $\mathrm{Re}(z) + \mathrm{Im}(z) \leq 3$

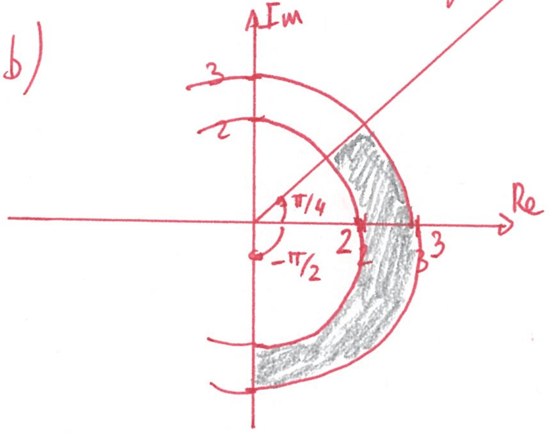
(e) the region common to $z\bar{z} \leq 4$ and $z + \bar{z} \leq 2$.

a) $|z-1| \leqslant 1$ is the disc centered on $(1,0)$, of radius 1

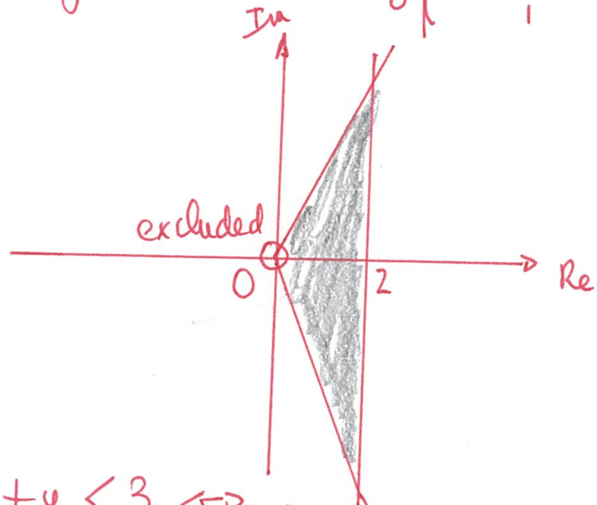
$\mathrm{Im}(z) \geqslant 0$ is the half-plane for which $y \geqslant 0$



b)

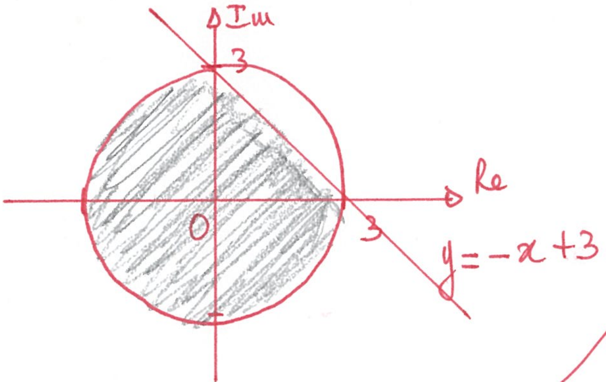


c)



d) $\mathrm{Re}(z) + \mathrm{Im}(z) \leqslant 3 \iff x + y \leqslant 3 \iff y \leqslant -x + 3$

whereas $|z| \leqslant 3$ is the disc centered on O, of radius 3



e) $z\bar{z} = (x+iy)(x-iy) = x^2 + y^2$

so $x^2 + y^2 \leqslant 4$

and $2x \leqslant 2$ or $x \leqslant 1$