INTEGRATION INVOLVING INVERSE TRIGONOMETRIC FUNCTIONS

2 Find the following.

(a) $\int \frac{dx}{\sqrt{16-x^2}}$ (b) $\int \frac{3}{9+x^2}dx$ (c) $\int \frac{dx}{\sqrt{1-x^2}}$ (d) $\int \frac{-1}{\sqrt{5-x^2}}dx$

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2 Find (i) $\int \frac{-1}{\sqrt{6-x^2}}dx$ (j) $\int \frac{dx}{\sqrt{1-4x^2}}$ (k) $\int \frac{dx}{1+9x^2}$ (l) $\int \frac{dx}{9+16x^2}$

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2 Find **(q)** $\int \frac{dx}{4+(x+5)^2}$ (let $u = x+5$) **(r)** $\int \frac{dx}{\sqrt{2-(x-3)^2}}$ (let $u = x-3$)

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3 Evaluate (k) $\int_0^1 \left(\frac{1}{1+x^2} + \frac{x}{1+x^2} \right) dx$ (m) $\int_{-4}^{4} \frac{dx}{x^2+16}$ (q) $\int_{-1}^{1} \frac{dx}{\sqrt{2-x^2}}$

6 On the same axes, sketch the graph of $y = 2\sin\frac{\pi x}{4}$ for $0 \le x \le 2$ and $x = 2\sin\frac{\pi y}{4}$ for $0 \le y \le 2$. Find the area of the region enclosed by the curves.

7 The curve $y=\frac{1}{\sqrt{1+x^2}}$ is rotated about the x-axis. Find the volume of the solid enclosed between $x=\frac{1}{\sqrt{3}}$ and $x=\sqrt{3}$.

9 Without evaluating the integral, explain why $\int_{-1}^{1} \tan^{-1} x\,dx$ is equal to zero.

12 Differentiate $x\cos^{-1} x - \sqrt{1-x^2}$ and use the result to evaluate $\int_{0}^{1} \cos^{-1} x\,dx$.

14 **(a)** Prove that $\frac{d}{dx}(x\sin^{-1}x)=\sin^{-1}x+\frac{x}{\sqrt{1-x^2}}$.

(b) Hence show that $\int_0^{\frac{1}{2}}\sin^{-1}x\,dx=\frac{\pi}{12}+\frac{\sqrt{3}}{2}-1$. (You may use the substitution $u=1-x^2$.)