SIMPLE LINEAR INEQUALITIES

The solution to an inequality is usually a range of numbers described by another inequality, related to the inequality in the question. The solution is also usually a range of real numbers, unless another set of numbers is specified in the question (e.g. integers).

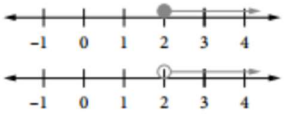
Rules for inequalities

If both sides of an inequality are multiplied or divided by a negative number, then the direction of the inequality is reversed.

If $a > b$, then:	If $a < b$, then:
$a + c > b + c$	$a + c < b + c$
$a - c > b - c$	$a - c < b - c$
$ac > bc$ if $c > 0$	$ac < bc$ if $c > 0$
$ac < bc$ if $c < 0$	$ac > bc$ if $c < 0$
$\frac{a}{c} > \frac{b}{c}$ if $c > 0$	$\frac{a}{c} < \frac{b}{c}$ if $c > 0$
$\frac{a}{c} < \frac{b}{c}$ if $c < 0$	$\frac{a}{c} > \frac{b}{c}$ if $c < 0$

On number lines:

- $a > b$ means that a is to the right of b.
- $a < b$ means that a is to the left of b.
- $x \geq 2$ is shown by a solid circle over 2 and an arrow to the right.
- $x > 2$ is shown by an empty circle over 2 and an arrow to the right.



This is demonstrated in the following examples.

Example 3

Solve $2x + 3 \geq 9$ and show the solution on a number line.

Solution

$2x + 3 \geq 9$

$2x \geq 6$

$x \geq 3$

SIMPLE LINEAR INEQUALITIES

Example 4

Solve $35 - 3x > 19 - x$ and show the solution on a number line, for the conditions:

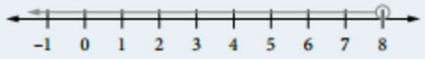
(a) x is a real number **(b)** x is an integer **(c)** x is not negative

Solution

$35 - 3x > 19 - x$

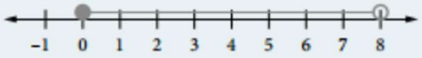
$35 > 19 + 2x$	Add $3x$ to both sides
$16 > 2x$	Subtract 19 from both sides
$8 > x$	Divide both sides by 2
$x < 8$	Rewrite starting with x

(a) For real numbers:
Solution is $x < 8$



(b) For integers:
Solution is $x = 7, 6, 5, \ldots$
(all integers to the left on the number line)

(c) For non-negative numbers, $x \geq 0$:
Solution is $0 \leq x < 8$



Example 5

Solve $\frac{x-1}{5} \leq \frac{x+1}{3}$ and show the solution on a number line.

Solution

$$\frac{x-1}{5} \leq \frac{x+1}{3}$$

$15 \times \frac{(x-1)}{5} \leq 15 \times \frac{(x+1)}{3}$	Multiply both sides by 15
$3(x-1) \leq 5(x+1)$	
$3x - 3 \leq 5x + 5$	Expand both sides
$-3 \leq 2x + 5$	Subtract $3x$ from both sides
$-8 \leq 2x$	Subtract 5 from both sides
$-4 \leq x$	Divide both sides by 2
$x \geq -4$	Rewrite starting with x

$-5 < 2x - 3 \leq 7$		or $2x - 3 > -5$	$2x - 3 \leq 7$
$-5 + 3 < 2x \leq 7 + 3$	Add 3 to each part	$2x > -5 + 3$	$2x \leq 7 + 3$
$-2 < 2x \leq 10$	Simplify	$2x > -2$	$2x \leq 10$
$-1 < x \leq 5$	Divide each part by 2	$x > -1$	$x \leq 5$

$-1 < x \leq 5$

The solution shows that x is greater than -1 but less than or equal to 5.