ARITHMETIC SEQUENCES

1 Which of the following are arithmetic sequences?

(a) 7, 17, 27, 37, ... (b) 5, 2, −1, −4, ... (c) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$

(d) $\frac{5}{8}, 1, 1\frac{3}{8}, 1\frac{3}{4}, \ldots$ (e) $\sqrt{2}-1, \sqrt{2}+1, \sqrt{2}+3, \sqrt{2}+5, \ldots$ (f) $\pi, \pi^2+1, \pi^3+2, \pi^4+3, \ldots$

a) YES, common difference is 10 b) YES, common difference is −3

b) NO, as $\left(\frac{1}{3}-\frac{1}{2}\right) \neq \left(\frac{1}{4}-\frac{1}{3}\right) \neq \left(\frac{1}{5}-\frac{1}{4}\right)$

d) $1-\frac{5}{8}=\frac{3}{8}$, $1\frac{3}{8}-1=\frac{3}{8}$ and $1\frac{3}{4}-1\frac{3}{8}=\frac{3}{8}$ so common difference is $\frac{3}{8}$ so YES

e) $(\sqrt{2}+1)-(\sqrt{2}-1)=2$; $(\sqrt{2}+3)-(\sqrt{2}+1)=2$, $(\sqrt{2}+5)-(\sqrt{2}+3)=2$ so common difference 2 so YES

f) Common difference is 1, so YES

2 For the arithmetic sequence 5, 8, 11, 14, ... find:

(a) the value of a (b) the value of d (c) the expression for T_n

(d) the 13th term (e) the value of k if $T_k = 98$.

a) a is the 1st term, here 5

b) d is the common difference, here 3

c) $T_n = 5 + 3(n-1) = 3n + 2$

d) $T_{13} = 5 + 3 \times (13-1) = 41$

e) $T_k = 98 = 5 + 3 \times (k-1)$

$$\text{so } (k-1) = \frac{98-5}{3} = 31$$

$$\therefore k = 32$$

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4 Find the eighth and fourteenth terms of the arithmetic sequence 8, 14, 20, 26, ...

1st term is 8, common difference is 6, $\therefore T_n = 8 + 6(n-1)$

$T_8 = 8 + 6 \times (8-1) = 50$

$T_{14} = 8 + 6 \times (14-1) = 86$

5 For the arithmetic sequence 17.2, 16.6, 16, 15.4, ... , find T_6 and T_{11}.

1st term is 17.2, common difference is -0.6 $\therefore T_n = 17.2 + (-0.6)(n-1)$

$T_6 = 17.2 - 0.6 \times (6-1) = 14.2$

$T_{11} = 17.2 - 0.6 \times (11-1) = 11.2$

6 The first and second terms of an arithmetic sequence are p and q respectively. Write an expression for the tenth term.

$T_n = p + (q-p)[n-1]$

$\therefore \quad T_{10} = p + (q-p) \times (10-1)$

$T_{10} = p + 9(q-p)$

$T_{10} = 9q - 8p$

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8 Find the arithmetic sequence in which $T_5 = 17$ and $T_{12} = 52$.

$$d = \frac{T_{12} - T_5}{12 - 5} = \frac{52 - 17}{7} = 5$$

$$\therefore T_n = T_1 + 5(n-1) \quad \text{so} \quad T_1 = T_n - 5(n-1) = T_n + 5 - 5n$$

For $n = 5$ $\quad T_1 = 17 + 5 - 5 \times 5 = -3$

$$T_n = -3 + 5 - 5n = 2 - 5n$$

9 Find T_6 of the arithmetic sequence in which $T_3 = 5.6$ and $T_{12} = -7$.

$$d = \frac{T_{12} - T_3}{12 - 3} = \frac{-7 - 5.6}{9} = -1.4$$

$$\therefore T_n = T_1 - 1.4(n-1) \quad \Leftrightarrow \quad T_1 = T_n + 1.4(n-1)$$

For $n = 3$ $\quad T_1 = T_3 + 1.4(3-1) = 5.6 + 2 \times 1.4 = 8.4$

$$\therefore T_n = 8.4 - 1.4(n-1) = -1.4n + 9.8$$

$$T_6 = -1.4 \times 6 + 9.8 = 1.4$$

11 Find the value of p so that $p + 5$, $4p + 3$, $8p - 2$ form the first three terms of an arithmetic sequence.

$$(4p + 3) - (p + 5) = 3p - 2$$

$$(8p - 2) - (4p + 3) = 4p - 5$$

The two must be equal for the sequence to be arithmetic,

$$\therefore \quad 3p - 2 = 4p - 5$$

$$\therefore \quad p = 3$$

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13 The first term of an arithmetic sequence is −8 and the seventh term is 22. Find the missing five terms of the sequence.

$T_1 = -8$, $T_7 = 22$ $\qquad d = \frac{T_7 - T_1}{7-1} = \frac{22-(-8)}{6} = 5$

$\therefore T_2 = -3$, $T_3 = 2$, $T_4 = 7$, $T_5 = 12$, $T_6 = 17$

15 Given 36, 31, 26, are the first three terms of an arithmetic sequence, find the value of n if the nth term is −4.

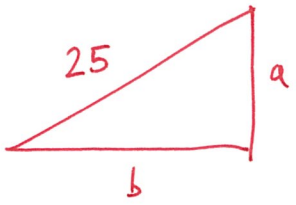
$d = -5$ $\qquad \therefore \; T_n = 36 - 5(n-1)$

if $T_k = -4$, then $-4 = 36 - 5(k-1)$

$\therefore \; 5(k-1) = 40$

$\therefore \; (k-1) = 8$ $\qquad \boxed{k = 9}$

17 The lengths of the sides of a right-angled triangle form the terms of an arithmetic sequence. If the hypotenuse is 25 cm long, find the lengths of the other two sides.



$a + d = b$ ①

and $b + d = 25$ ②

and $25 = a^2 + b^2$ ③

① + ② $\Rightarrow$ $a + b + 2d = 25 + b$ so $a + 2d = 25$ so $b =$

③ becomes $25^2 = (25 - 2d)^2 + (25 - d)^2$

so $25^2 = 625 - 100d + 4d^2 + 625 - 50d + d^2$

$\therefore \; 5d^2 - 150d + 625 = 0$ $\qquad \therefore \; d^2 - 30d + 125 = 0$

$d = \frac{30 \pm 20}{2}$ so $d = 5$ or $d = 25$ (impossible) so $b = 20$

and $a = 15$

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19 Alexandra starts a new job with an initial salary of \$28 000 per annum. She is promised an increase of \$300 per quarter for the first 4 years of her employment.

(a) What will her salary be in 3 months time?
(b) Write an expression for her salary, S, in dollars after n quarters.
(c) What is the domain of the function $S(n)$?
(d) What is the maximum salary that she can expect to receive?

a) $28000 + 300 = 28{,}300$

b) $S_n = 28{,}000 + n \times 300$

c) The relation $S_n = 28{,}000 + 300n$ is only valid for the first 4 years of employment, so $4 \times 4 = 16$ quarters

$n \leq 16$.

d) $S_{16} = 28{,}000 + 300 \times 16$

$S_{16} = 32{,}800$

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20 $1000 is borrowed for two years at a simple interest rate of 0.5% per month, with the interest added monthly.

(a) How much interest is added each month?
(b) How much is owed after 1 month?
(c) How much is owed after one year?
(d) How much is to be paid back at the end of two years?

a) $I = 1{,}000 \times 0.005 = 5$

b) $1{,}000 + 5 = 1{,}005$ is owned after 1 month

c) $1{,}000 + 5 \times 12 = 1{,}060$ is owed after 1 year

d) $1{,}000 + 5 \times 24 = 1{,}120$ is to be paid back at the end of two years.