USING IDENTITIES TO SIMPLIFY EXPRESSIONS AND PROVE RESULTS

Prove the following identities (questions **1** to **21**):

1 $\dfrac{\sin A+\cos A\tan B}{\cos A-\sin A\tan B}=\tan(A+B)$

2 $\dfrac{\sin 2\theta\cos\theta-\cos 2\theta\sin\theta}{\cos 2\theta\cos\theta+\sin 2\theta\sin\theta}=\tan\theta$

3 $\dfrac{\tan A-\tan B}{\tan A+\tan B}=\dfrac{\sin(A-B)}{\sin(A+B)}$

4 $\sin(\theta+\alpha)\sin(\theta-\alpha)=\sin^2\theta-\sin^2\alpha$

1) $\dfrac{\sin A+\cos A\tan B}{\cos A-\sin A\tan B}=\dfrac{\dfrac{\sin A+\cos A\tan B}{\cos A}}{\dfrac{\cos A-\sin A\tan B}{\cos A}}=\dfrac{\tan A+\tan B}{1-\tan A\tan B}=\tan(A+B)$

2) $\dfrac{\sin 2\theta\cos\theta-\cos 2\theta\sin\theta}{\cos 2\theta\cos\theta+\sin 2\theta\sin\theta}=\dfrac{\sin(2\theta-\theta)}{\cos(2\theta-\theta)}=\dfrac{\sin\theta}{\cos\theta}=\tan\theta$

3) $\dfrac{\tan A-\tan B}{\tan A+\tan B}=\dfrac{\left(\dfrac{\sin A}{\cos A}-\dfrac{\sin B}{\cos B}\right)}{\left(\dfrac{\sin A}{\cos A}+\dfrac{\sin B}{\cos B}\right)}=\dfrac{\left(\dfrac{\sin A}{\cos A}-\dfrac{\sin B}{\cos B}\right)}{\left(\dfrac{\sin A}{\cos A}+\dfrac{\sin B}{\cos B}\right)}\times\dfrac{\cos A\cos B}{\cos A\cos B}$

$\_\_\_\_ = \dfrac{\sin A\cos B-\sin B\cos A}{\sin A\cos B+\cos A\sin B}=\dfrac{\sin(A-B)}{\sin(A+B)}$

4) $\sin(\theta+\alpha)\sin(\theta-\alpha)=[\sin\theta\cos\alpha+\cos\theta\sin\alpha][\sin\theta\cos\alpha-\cos\theta\sin\alpha]$

$\_\_\_\_ = (\sin\theta\cos\alpha)^2-\sin\theta\cos\theta\sin\alpha\cos\alpha+\sin\theta\cos\theta\sin\alpha\cos\alpha-(\cos\theta\sin\alpha)^2$

$\_\_\_\_ = (\sin\theta\cos\alpha)^2-(\cos\theta\sin\alpha)^2$

$\_\_\_\_ = \sin^2\theta\cos^2\alpha-\cos^2\theta\sin^2\alpha$

but $\cos^2\alpha=1-\sin^2\alpha$ and $\cos^2\theta=1-\sin^2\theta$

$\_\_\_\_ = \sin^2\theta(1-\sin^2\alpha)-(1-\sin^2\theta)\sin^2\alpha$

$\_\_\_\_ = \sin^2\theta-\sin^2\theta\sin^2\alpha-\sin^2\alpha+\sin^2\theta\sin^2\alpha$

$\_\_\_\_ = \sin^2\theta-\sin^2\alpha$

USING IDENTITIES TO SIMPLIFY EXPRESSIONS AND PROVE RESULTS

Prove the following identities (questions 1 to 21):

9 $\dfrac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}+\dfrac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}=2\sec 2\theta$

10 $\dfrac{1-\cos x}{\sin x}=\tan\dfrac{x}{2}$

11 $\dfrac{\sin A+\sin(90^\circ-A)+1}{\sin A-\sin(90^\circ-A)+1}=\cot\dfrac{A}{2}$

12 $\dfrac{\sin x+1-\cos x}{\sin x-1+\cos x}=\dfrac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}$

9) $$\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}+\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}=\frac{(\cos\theta+\sin\theta)^2+(\cos\theta-\sin\theta)^2}{(\cos\theta-\sin\theta)\times(\cos\theta+\sin\theta)}$$

$$\underline{\qquad}=\frac{2\cos\theta\sin\theta+1+1-2\cos\theta\sin\theta}{\cos^2\theta-\sin^2\theta}=\frac{2}{\cos 2\theta}=2\sec 2\theta$$

10) $$\frac{1-\cos x}{\sin x}=\frac{2\sin^2(x/2)}{2\sin(x/2)\cos(x/2)}=\frac{\sin(x/2)}{\cos(x/2)}=\tan(x/2)$$

11) $$\frac{\sin A+\sin(90-A)+1}{\sin A-\sin(90-A)+1}=\frac{[2\sin(A/2)\cos(A/2)]+[2\cos^2(A/2)-1]+1}{[2\sin(A/2)\cos(A/2)]-[1-2\sin^2(A/2)]+1}$$

$$\underline{\qquad}=\frac{\sin(A/2)\cos(A/2)+\cos^2(A/2)}{\sin(A/2)\cos(A/2)+\sin^2(A/2)}=\frac{\cos(A/2)\,[\sin(A/2)+\cos(A/2)]}{\sin(A/2)\,[\sin(A/2)+\cos(A/2)]}$$

$$\underline{\qquad}=\frac{\cos(A/2)}{\sin(A/2)}=\operatorname{cotan}\left(\frac{A}{2}\right)$$

12) $$\frac{\sin x+1-\cos x}{\sin x-1+\cos x}=\frac{\dfrac{\sin x+1-\cos x}{\sin x}}{\dfrac{\sin x-1+\cos x}{\sin x}}=\frac{1+\left(\dfrac{1-\cos x}{\sin x}\right)}{1-\left(\dfrac{1-\cos x}{\sin x}\right)}$$

$$\underline{\qquad}=\frac{1+\left(\dfrac{2\sin^2(x/2)}{2\sin(x/2)\cos(x/2)}\right)}{1-\left(\dfrac{2\sin^2(x/2)}{2\sin(x/2)\cos(x/2)}\right)}$$

$$\underline{\qquad}=\frac{1+\dfrac{\sin(x/2)}{\cos(x/2)}}{1-\dfrac{\sin(x/2)}{\cos(x/2)}}=\frac{1+\tan(x/2)}{1-\tan(x/2)}$$

USING IDENTITIES TO SIMPLIFY EXPRESSIONS AND PROVE RESULTS

14 $\cos(A+B+C) = \cos A\cos B\cos C - \cos A\sin B\sin C - \cos B\sin C\sin A - \cos C\sin A\sin B$

What is the resulting identity if B is replaced by $(90° - C)$?

$$\cos(A+(90-C)+C) = \cos A\cos(90-C)\cos C - \cos A\sin(90-C)\sin C - \cos(90-C)\sin C\sin A - \cos C\sin A\sin(90-C)$$

$$\cos(A+90) = \cos A\cancel{\sin C}\cos C - \cos A\cancel{\cos C}\sin C - \sin C\sin C\sin A - \cos C\sin A\cos C$$

$$\cos(A+90) = -\sin A\left(\sin^2 C + \cos^2 C\right)$$

So $\cos(A+90) = -\sin A$.

21 $\dfrac{1-\tan\theta\tan 2\theta}{1+\tan\theta\tan 2\theta} = 4\cos^2\theta - 3$

$$\frac{1-\tan\theta\tan 2\theta}{1+\tan\theta\tan 2\theta} = \frac{1-\tan\theta\times\dfrac{2\tan\theta}{1-\tan^2\theta}}{1+\tan\theta\times\dfrac{2\tan\theta}{1-\tan^2\theta}}$$

$$\_\_\_ = \frac{1-\dfrac{2\tan^2\theta}{1-\tan^2\theta}}{1+\dfrac{2\tan^2\theta}{1-\tan^2\theta}}$$

$$\_\_\_ = \frac{(1-\tan^2\theta)-2\tan^2\theta}{(1-\tan^2\theta)+2\tan^2\theta} = \frac{1-3\tan^2\theta}{1+\tan^2\theta}$$

$$\_\_\_ = \frac{1-3\tan^2\theta}{\sec^2\theta} = \frac{1}{\sec^2\theta} - 3\,\frac{\dfrac{\sin^2\theta}{\cos^2\theta}}{\dfrac{1}{\cos^2\theta}} = \cos^2\theta - 3\sin^2\theta$$

$$\_\_\_ = \cos^2\theta - 3(1-\cos^2\theta) = 4\cos^2\theta - 3$$

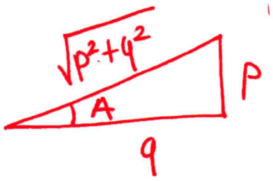
USING IDENTITIES TO SIMPLIFY EXPRESSIONS AND PROVE RESULTS

22 If $\tan A = \frac{p}{q}$, express the following in terms of p and q.

(a) $q\sin A\cos A + p\sin^2 A$ **(b)** $p\sin 2A + q\cos 2A$

a) $q\sin A\cos A + p\sin^2 A = q \times \frac{p}{\sqrt{p^2+q^2}} \times \frac{q}{\sqrt{p^2+q^2}} + p \times \frac{p^2}{p^2+q^2}$

$= \frac{pq^2 + p \times p^2}{p^2+q^2} = p\,\frac{(p^2+q^2)}{p^2+q^2} = p$



b) $p\sin 2A + q\cos 2A = 2p\sin A\cos A + q(2\cos^2 A - 1)$

but $\sin A = \frac{p}{\sqrt{p^2+q^2}}$ and $\cos A = \frac{q}{\sqrt{p^2+q^2}}$

So $p\sin 2A + q\cos 2A = 2p \times \frac{p}{\sqrt{p^2+q^2}} \times \frac{q}{\sqrt{p^2+q^2}} + q\left[\frac{2q^2}{(\sqrt{p^2+q^2})^2} - 1\right]$

$= \frac{2p^2q}{(p^2+q^2)} + q\left[\frac{2q^2 - (p^2+q^2)}{p^2+q^2}\right]$

$= \frac{2p^2q + q \times (q^2 - p^2)}{p^2+q^2}$

$= \frac{p^2q + q \times q^2}{p^2+q^2}$

$= \frac{q(p^2+q^2)}{p^2+q^2} = q$

USING IDENTITIES TO SIMPLIFY EXPRESSIONS AND PROVE RESULTS

23 If A, B and C are the angles of a triangle, prove that $\cos A \cos B - \sin A \sin B + \cos C = 0$.

$$\cos A \cos B - \sin A \sin B + \cos C = \cos(A+B) + \cos C$$

$$= \cos(A+B) + \cos(180 - (A+B))$$

as $A+B+C = 180°$ (sum of angles of triangle)

$$= \cos(A+B) + (-\cos(A+B))$$

$$= 0$$

24 Given that $\sin 18° = \frac{1}{4}(\sqrt{5}-1)$, find $\cos 36°$ in surd form.

$$\cos 36 = \cos(2 \times 18) = 1 - 2 \times \sin^2(18)$$

$$\cos 36 = 1 - 2 \times \left[\frac{1}{4}(\sqrt{5}-1)\right]^2$$

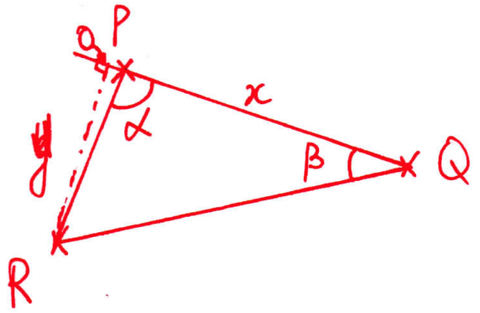
$$\cos 36 = 1 - \frac{2}{16} \times [5 - 2\sqrt{5} + 1] = 1 - \frac{1}{8}[6 - 2\sqrt{5}]$$

$$\cos 36 = 1 - \frac{6}{8} + \frac{1}{4}\sqrt{5}$$

$$\cos 36 = \frac{1+\sqrt{5}}{4}$$

USING IDENTITIES TO SIMPLIFY EXPRESSIONS AND PROVE RESULTS

26 Three points P, Q, R are in a horizontal plane. Angles RPQ and RQP are α and β respectively. If PQ is x units in length, show that the perpendicular distance y from R to PQ is given by $y = \dfrac{x\tan\alpha\tan\beta}{\tan\alpha + \tan\beta}$.



$$\tan(180 - \alpha) = \frac{y}{OP}$$

but $\tan(180-\alpha) = \dfrac{\sin(180-\alpha)}{\cos(180-\alpha)} = -\tan\alpha$

so $y = -OP\tan\alpha$ or $OP = \dfrac{-y}{\tan\alpha}$

Further $\tan\beta = \dfrac{y}{OQ}$ so $y = OQ\tan\beta$ or $OQ = \dfrac{y}{\tan\beta}$

Now $x = OQ - OP = \dfrac{y}{\tan\beta} - \left(\dfrac{-y}{\tan\alpha}\right)$

So $x = \dfrac{y}{\tan\beta} + \dfrac{y}{\tan\alpha} = y\left[\dfrac{1}{\tan\beta} + \dfrac{1}{\tan\alpha}\right]$

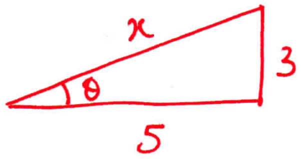
$$x = y\left[\frac{\tan\alpha + \tan\beta}{\tan\alpha\tan\beta}\right]$$

$$\therefore\ y = x\frac{\tan\alpha\tan\beta}{\tan\alpha + \tan\beta}$$

USING IDENTITIES TO SIMPLIFY EXPRESSIONS AND PROVE RESULTS

$180^\circ < \theta < 270^\circ$ so III quadrant $\sin\theta < 0$ and $\cos\theta < 0$

29 If $\tan\theta = \frac{3}{5}$ and $\pi < \theta < \frac{3\pi}{2}$, find the value of: **(a)** $\sin\theta$ **(b)** $\cos\theta$ **(c)** $\cos 2\theta$



$$x^2 = 3^2 + 5^2 = 9 + 25 = 34$$

so $x = \sqrt{34}$

so $\sin\theta = -\dfrac{3}{\sqrt{34}}$ and $\cos\theta = -\dfrac{5}{\sqrt{34}}$

c) $\cos 2\theta = 2\cos^2\theta - 1$

$$= 2 \times \left(\frac{-5}{\sqrt{34}}\right)^2 - 1$$

$$= \frac{50}{34} - 1 = \frac{16}{34} = \frac{8}{17}$$

$180 < \alpha < 270$ III quadrant so both $\sin\alpha$ and $\cos\alpha$ are negative.

31 If $\operatorname{cosec}\alpha = -\frac{17}{8}$ and $\pi < \alpha < \frac{3\pi}{2}$, find the value of: **(a)** $\cot\alpha$ **(b)** $\tan 2\alpha$

$\operatorname{cosec}\alpha = -\dfrac{17}{8} = \dfrac{1}{\sin\alpha}$ so $\sin\alpha = -\dfrac{8}{17}$

b) $\operatorname{cosec}^2\alpha = 1 + \cot^2\alpha$ so $\cot^2\alpha = \operatorname{cosec}^2\alpha - 1$

$\operatorname{cotan}^2\alpha = \left(\dfrac{17}{8}\right)^2 - 1 = \dfrac{225}{64}$ so $\operatorname{cotan}\alpha = \dfrac{15}{8}$ (positive as both $\sin\alpha$ and $\cos\alpha$ are negative)

c) $\tan 2\alpha = \dfrac{2\tan\alpha}{1 - \tan^2\alpha}$ But $\operatorname{cotan}\alpha = \dfrac{15}{8}$ so $\tan\alpha = \dfrac{8}{15}$

$$\tan 2\alpha = \frac{2 \times \frac{8}{15}}{1 - \left(\frac{8}{15}\right)^2} = \frac{\frac{16}{15}}{\frac{161}{225}} = \frac{16}{15} \div \frac{161}{225} = \frac{16}{15} \times \frac{225}{161} = \frac{240}{161}$$

USING IDENTITIES TO SIMPLIFY EXPRESSIONS AND PROVE RESULTS

37 **(a)** By writing expansions for $\sin(A+B)$ and $\sin(A-B)$, find a simplified expression for $\sin(A+B)+\sin(A-B)$.

(b) By writing $\theta = A+B$ and $\phi = A-B$, find an expression for $\sin\theta + \sin\phi$ as the product of two trigonometric functions.

a) $\sin(A+B) + \sin(A-B) = \sin A\cos B + \cos A\sin B + \sin A\cos B - \cos A\sin B$

$= 2\sin A\cos B$

b) If $\begin{cases}\theta = A+B \\ \phi = A-B\end{cases}$ then $\begin{cases}2A = \theta+\phi \\ 2B = \theta-\phi\end{cases}$ so $\begin{cases}A = \dfrac{\theta+\phi}{2} \\ B = \dfrac{\theta-\phi}{2}\end{cases}$

So $\sin\theta + \sin\phi = 2\sin\left(\dfrac{\theta+\phi}{2}\right)\cos\left(\dfrac{\theta-\phi}{2}\right)$

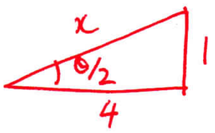
38 If $\sec\theta - \tan\theta = \frac{3}{5}$, show that $\sin\theta = \frac{8}{17}$. (Hint: Use t formulae.) with $t = \tan\left(\frac{\theta}{2}\right)$

$$\sec\theta - \tan\theta = \frac{1}{\cos\theta} - \tan\theta = \frac{1-\sin\theta}{\cos\theta} = \frac{1-\frac{2t}{1+t^2}}{\frac{1-t^2}{1+t^2}} = \frac{1-2t+t^2}{1-t^2}$$

$$= \frac{(1-t)^2}{(1-t)(1+t)} = \frac{1-t}{1+t} = \frac{3}{5}$$

So $5(1-t) = 3(1+t) \iff -5t+5 = 3+3t \iff 8t = 2$ so $t = 1/4$

So $\tan\left(\frac{\theta}{2}\right) = \frac{1}{4}$



$x^2 = 1^2 + 4^2 = 17$ so $x = \sqrt{17}$

$\sin\left(\frac{\theta}{2}\right) = \frac{1}{\sqrt{17}}$ and $\cos\left(\frac{\theta}{2}\right) = \frac{4}{\sqrt{17}}$

so $\sin\theta = 2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right) = 2 \times \frac{1}{\sqrt{17}} \times \frac{4}{\sqrt{17}} = \frac{8}{17}$

39 If $4\tan(\alpha-\beta)=3\tan\alpha$, prove that $\tan\beta=\dfrac{\sin 2\alpha}{7+\cos 2\alpha}$.

$$4\tan(\alpha-\beta)=4\times\left[\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}\right]=3\tan\alpha$$

so $4[\tan\alpha-\tan\beta]=3\tan\alpha\,[1+\tan\alpha\tan\beta]$

$\Leftrightarrow\; 4\tan\alpha-4\tan\beta=3\tan\alpha+3\tan^2\alpha\tan\beta$

$\Leftrightarrow\; \tan\beta\times[-4-3\tan^2\alpha]=3\tan\alpha-4\tan\alpha=-\tan\alpha.$

$$\text{So}\quad \tan\beta=\frac{\tan\alpha}{4+3\tan^2\alpha}=\frac{\frac{\sin\alpha}{\cos\alpha}}{4+3\frac{\sin^2\alpha}{\cos^2\alpha}}\times\frac{\cos^2\alpha}{\cos^2\alpha}$$

$$\tan\beta=\frac{\sin\alpha\cos\alpha}{4\cos^2\alpha+3\sin^2\alpha}$$

$$\tan\beta=\frac{2\sin\alpha\cos\alpha}{2\left[4\cos^2\alpha+3(1-\cos^2\alpha)\right]}$$

$$\tan\beta=\frac{\sin 2\alpha}{2[\cos^2\alpha+3]}$$

$$\tan\beta=\frac{\sin 2\alpha}{2\cos^2\alpha+6}$$

but $\cos 2\theta=2\cos^2\theta-1$

so $2\cos^2\theta=\cos 2\theta+1$

$$\therefore\; \tan\beta=\frac{\sin 2\alpha}{\cos 2\alpha+1+6}=\frac{\sin 2\alpha}{7+\cos 2\alpha}$$

USING IDENTITIES TO SIMPLIFY EXPRESSIONS AND PROVE RESULTS

40 Use the factors of $x^3 - y^3$ to show that $\cos^6\theta - \sin^6\theta = \left(1 - \frac{1}{4}\sin^2 2\theta\right)\cos 2\theta$.

$$\cos^6\theta - \sin^6\theta = (\cos^2\theta)^3 - (\sin^2\theta)^3 \qquad \text{but } a^3 - b^3 = (a-b)(a^2+ab+b^2)$$

$$= \left[\cos^2\theta - \sin^2\theta\right]\left[(\cos^2\theta)^2 + (\cos^2\theta)(\sin^2\theta) + (\sin^2\theta)^2\right]$$

$$= \cos 2\theta\left[\cos^4\theta + (\cos\theta\sin\theta)^2 + \sin^4\theta\right]$$

$$= \cos 2\theta\left[\cos^2\theta \times \cos^2\theta + \left(\frac{\sin 2\theta}{2}\right)^2 + \sin^2\theta \times \sin^2\theta\right]$$

$$= \cos 2\theta\left[\cos^2\theta \times (1 - \sin^2\theta) + \frac{1}{4}\sin^2 2\theta + \sin^2\theta(1 - \cos^2\theta)\right]$$

$$= \cos 2\theta\left[\cos^2\theta + \sin^2\theta - \sin^2\theta\cos^2\theta + \frac{1}{4}\sin^2 2\theta - \sin^2\theta\cos^2\theta\right]$$

$$= \cos 2\theta\left[1 - 2\sin^2\theta\cos^2\theta + \frac{1}{4}\sin^2 2\theta\right]$$

$$= \cos 2\theta\left[1 - \frac{1}{2} \times (2\sin\theta\cos\theta)^2 + \frac{1}{4}\sin^2 2\theta\right]$$

$$= \cos 2\theta\left[1 - \frac{1}{2}\sin^2 2\theta + \frac{1}{4}\sin^2 2\theta\right]$$

$$= \cos 2\theta\left[1 - \frac{1}{4}\sin^2 2\theta\right]$$

USING IDENTITIES TO SIMPLIFY EXPRESSIONS AND PROVE RESULTS

41 If $\tan\theta = t$, express $\sin 2\theta$ and $\cos 2\theta$ in terms of t. Find the values of t for which $(k+1)\sin 2\theta + (k-1)\cos 2\theta = k+1$.

if $\tan\left(\frac{\theta}{2}\right) = t$ then $\sin\theta = \frac{2t}{1+t^2}$ and $\cos\theta = \frac{1-t^2}{1+t^2}$

So if $\tan\theta = t$ then $\sin 2\theta = \frac{2t}{1+t^2}$ and $\cos 2\theta = \frac{1-t^2}{1+t^2}$

$$(k+1)\sin 2\theta + (k-1)\cos 2\theta = (k+1)\times\frac{2t}{1+t^2} + (k-1)\times\frac{(1-t^2)}{1+t^2}$$

$$= \frac{(k+1)\times 2t + (k-1)\times(1-t^2)}{1+t^2}$$

For this to be equal to $(k+1)$ we must have:

$$(k+1)2t + (k-1)(1-t^2) = (k+1)(1+t^2)$$

i.e: $t^2\left[-k+1-k-1\right] + t\left[2(k+1)\right] + \left[k-1-k-1\right] = 0$

$$\Leftrightarrow \quad -2k\,t^2 + 2(k+1)t - 2 = 0$$

$$\Leftrightarrow \quad k t^2 - (k+1)t + 1 = 0$$

$$\Delta = (k+1)^2 - 4k = k^2 + 2k + 1 - 4k = k^2 - 2k + 1 = (k-1)^2$$

So $t = \frac{(k+1)-(k-1)}{2k} = \frac{2}{2k} = \frac{1}{k}$

or $t = \frac{(k+1)+(k-1)}{2k} = \frac{2k}{2k} = 1$

two possible values of t, 1 or $\frac{1}{k}$

44 If $\tan\alpha = k\tan\beta$, show that $(k-1)\sin(\alpha+\beta) = (k+1)\sin(\alpha-\beta)$.

Proving $(k-1)\sin(\alpha+\beta) = (k+1)\sin(\alpha-\beta)$ is the same than proving that $\dfrac{(k-1)\sin(\alpha+\beta)}{(k+1)\sin(\alpha-\beta)} = 1$

$$\frac{(k-1)\sin(\alpha+\beta)}{(k+1)\sin(\alpha-\beta)} = \frac{(k-1)\left[\sin\alpha\cos\beta + \cos\alpha\sin\beta\right]}{(k+1)\left[\sin\alpha\cos\beta - \cos\alpha\sin\beta\right]}$$

$$\text{———} = \frac{k-1}{k+1} \times \left[\frac{\dfrac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\sin\beta}}{\dfrac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\cos\alpha\sin\beta}}\right]$$

$$\text{———} = \frac{k-1}{k+1} \times \left[\frac{\dfrac{\tan\alpha}{\tan\beta} + 1}{\dfrac{\tan\alpha}{\tan\beta} - 1}\right]$$

but $\dfrac{\tan\alpha}{\tan\beta} = k$, so

$$\text{———} = \frac{k-1}{k+1} \times \left[\frac{k+1}{k-1}\right] = 1$$

So indeed $\dfrac{(k-1)\sin(\alpha+\beta)}{(k+1)\sin(\alpha-\beta)} = 1$

$\therefore\ (k-1)\sin(\alpha+\beta) = (k+1)\sin(\alpha-\beta)$.