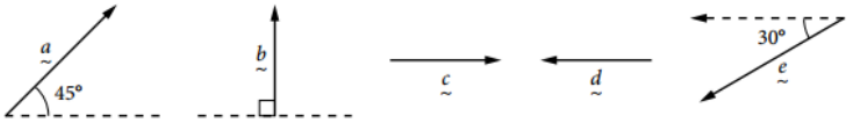
SCALAR PRODUCT OF VECTORS

1 Consider the vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$, $\underset{\sim}{c}$, $\underset{\sim}{d}$ and $\underset{\sim}{e}$ as shown.



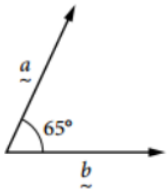
Find the angle between the following pairs of vectors.

(a) $\underset{\sim}{a}$ and $\underset{\sim}{b}$ **(b)** $\underset{\sim}{a}$ and $\underset{\sim}{c}$ **(c)** $\underset{\sim}{a}$ and $\underset{\sim}{d}$ **(d)** $\underset{\sim}{a}$ and $\underset{\sim}{e}$ **(e)** $\underset{\sim}{b}$ and $\underset{\sim}{c}$

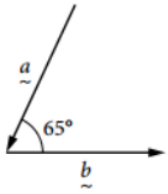
(f) $\underset{\sim}{b}$ and $\underset{\sim}{d}$ **(g)** $\underset{\sim}{b}$ and $\underset{\sim}{e}$ **(h)** $\underset{\sim}{c}$ and $\underset{\sim}{d}$ **(i)** $\underset{\sim}{c}$ and $\underset{\sim}{e}$ **(j)** $\underset{\sim}{d}$ and $\underset{\sim}{e}$

2 Given $|\underset{\sim}{a}| = 8$ and $|\underset{\sim}{b}| = 7$, find the scalar product of $\underset{\sim}{a}$ and $\underset{\sim}{b}$ for each of the following, correct to two decimal places where necessary.

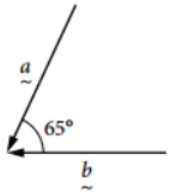
(a)



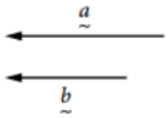
(b)



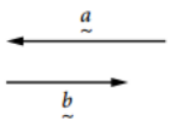
(c)



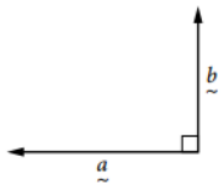
(d)



(e)



(f)



4 Show that the vectors $\underset{\sim}{a} = 3\underset{\sim}{i} + 7\underset{\sim}{j}$ and $\underset{\sim}{b} = 7\underset{\sim}{i} - 3\underset{\sim}{j}$ are perpendicular.

5 Find the vector $\underset{\sim}{d}$ that is perpendicular to $\underset{\sim}{c} = 4\underset{\sim}{i} - 3\underset{\sim}{j}$ and has a magnitude of 10.

6 If the vectors $\underset{\sim}{e} = 7\underset{\sim}{i} - 5\underset{\sim}{j}$ and $\underset{\sim}{f} = x\underset{\sim}{i} - 3\underset{\sim}{j}$ are perpendicular, find the value of x.

7 If $\underset{\sim}{a} = -6\underset{\sim}{i} + 2\underset{\sim}{j}$, find: **(a)** $\underset{\sim}{a} \bullet \underset{\sim}{a}$ **(b)** $|\underset{\sim}{a}|$ **(c)** $\underset{\sim}{a} \bullet \underset{\sim}{a}$ in terms of $|\underset{\sim}{a}|$

8 For any vector $\underset{\sim}{a}$, find the value of each of the following, in terms of $|a|$ where necessary.

(a) $\underset{\sim}{a} \bullet \underset{\sim}{a}$ **(b)** $\hat{\underset{\sim}{a}} \bullet \hat{\underset{\sim}{a}}$ **(c)** $\underset{\sim}{a} \bullet (-\underset{\sim}{a})$

10 Find the angle, correct to the nearest degree, between each of the following pairs of vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$:

(a) $\underset{\sim}{a} = 3\underset{\sim}{i} + 2\underset{\sim}{j}$ and $\underset{\sim}{b} = 3\underset{\sim}{i} + 5\underset{\sim}{j}$ **(b)** $\underset{\sim}{a} = -3\underset{\sim}{i} + 2\underset{\sim}{j}$ and $\underset{\sim}{b} = 5\underset{\sim}{i} + 6\underset{\sim}{j}$ **(c)** $\underset{\sim}{a} = 4\underset{\sim}{i} - \underset{\sim}{j}$ and $\underset{\sim}{b} = 3\underset{\sim}{i} + 4\underset{\sim}{j}$

11 Which vector is perpendicular to $\underset{\sim}{f} = -5\underset{\sim}{i} + 2\underset{\sim}{j}$ with magnitude 12?

A $\underset{\sim}{a} = \frac{12}{\sqrt{29}}\left(5\underset{\sim}{i} - 2\underset{\sim}{j}\right)$ **B** $\underset{\sim}{b} = \frac{12}{\sqrt{29}}\left(2\underset{\sim}{i} + 5\underset{\sim}{j}\right)$ **C** $\underset{\sim}{c} = \frac{12}{\sqrt{29}}\left(2\underset{\sim}{i} - 5\underset{\sim}{j}\right)$ **D** $\underset{\sim}{d} = \frac{12}{\sqrt{29}}\left(-2\underset{\sim}{i} + 5\underset{\sim}{j}\right)$

12 Vectors $\underset{\sim}{a} = x\underset{\sim}{i} - 2\underset{\sim}{j}$ and $\underset{\sim}{b} = -6\underset{\sim}{i} + y\underset{\sim}{j}$ are perpendicular. What are possible values of x and y?

A $x = 1$ and $y = 3$ **B** $x = 1$ and $y = -3$ **C** $x = -2$ and $y = -6$ **D** $x = 2$ and $y = 6$

14 The points A, B and C have position vectors $\overrightarrow{OA} = -2\underset{\sim}{i} - 3\underset{\sim}{j}$, $\overrightarrow{OB} = 2\underset{\sim}{i} + 3\underset{\sim}{j}$ and $\overrightarrow{OC} = 8\underset{\sim}{i} - \underset{\sim}{j}$.

(a) Find the vectors $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{AC}$ in component form.

(b) Find $\left|\overrightarrow{AB}\right|$, $\left|\overrightarrow{BC}\right|$ and $\left|\overrightarrow{AC}\right|$.

(c) Show that ΔABC is a right-angled triangle.

(d) Find the position vector of a point D such that $ABCD$ forms a square.

(e) Find the vector $\overrightarrow{BD}$, the other diagonal of the square $ABCD$.

(f) Show that the diagonals of the square $ABCD$ bisect at right angles.