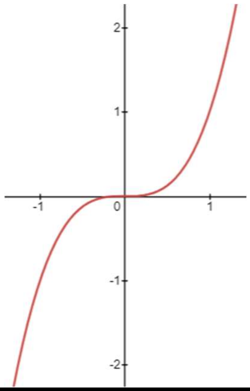
CONTINUITY AND SMOOTHNESS OF A FUNCTION OVER AN INTERVAL

Below is the graph of $f(x) = x^3$ (a cubic)



This graph is **continuous** as it has no gap or jump.

Formal definition of continuity:

A function f that is defined in some region that includes $x = c$ is said to be continuous at c if:

a) the function has a definite value $f(c)$ at c; and

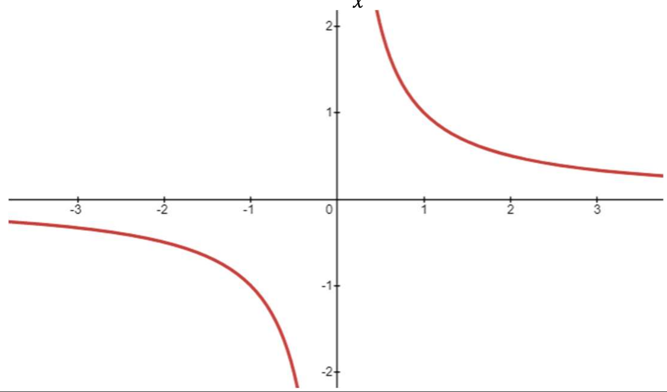
b) as $x \to c$, then $f(x) \to f(c)$

In Maths notation, this is noted:

$$\lim_{x \to c} f(x) = f(c)$$

A function is said to be continuous on the interval $[a, b]$ if its graph can be drawn from $x = a$ to $x = b$ without lifting the pencil off the paper.

Below is the graph of $f(x) = \frac{1}{x}$ (a hyperbola)



This graph is **discontinuous** as it has a gap (at $x = 0$ for this particular graph).

Further we note that when x becomes very large (i.e. "tends towards $+\infty$"), then $f(x)$ tends towards 0; in Mathematical notation, this is noted:

$\lim_{x \to +\infty} f(x) = 0$ we say: "*the **limit** of $f(x) = \frac{1}{x}$ when x tends towards $+\infty$ is zero*"

Likewise, for this particular curve, we can also say:

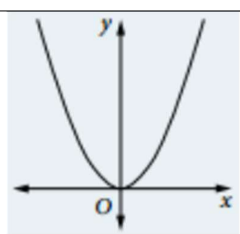
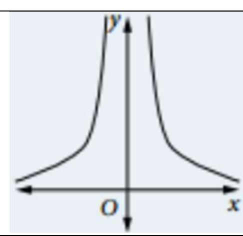
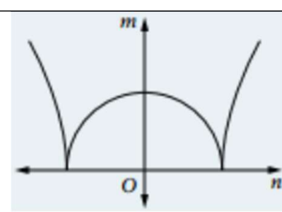
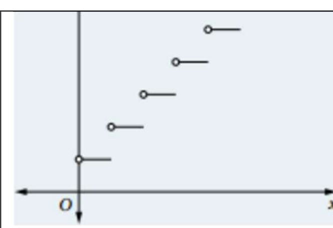
$$\lim_{x \to -\infty} f(x) = 0$$

Further:

$\lim_{x \to 0^+} f(x) = +\infty$ (i.e., when x tends towards 0, but approaching from the right-hand side (positive values), then $f(x)$ becomes very large (i.e. then $f(x)$ tends towards $+\infty$).

$\lim_{x \to 0^-} f(x) = -\infty$ (i.e., when x tends towards 0, but approaching from the left-hand side (negative values), then $f(x)$ becomes very small (i.e. then $f(x)$ tends towards -∞).

A function is said to be **smooth over an interval** if its slope does not make sharp corners, over that interval.

			
• continuous over $\mathbb{R}$ • smooth over $\mathbb{R}$	• continuous over $\mathbb{R}^+$ and also over $\mathbb{R}^-$, but discontinuous over $\mathbb{R}$ • smooth over $\mathbb{R}^+$ and also over $\mathbb{R}^-$, excluding 0	• continuous over $\mathbb{R}$ • not smooth at two points	• continuous over small intervals, but discontinuous over $\mathbb{R}$ • not smooth over $\mathbb{R}$

Note that if a function is discontinuous over an interval, then it is not smooth over that interval.