QUADRATIC FUNCTIONS

1 The minimum value of $x^2 - 2x + 6$ is:

A -1 B 1 $\boxed{\text{C}}$ 5 D 6

$y = (x-1)^2 - 1 + 6 = (x-1)^2 + 5$

2 Express each of the following functions in the form $y = a(x+B)^2 + C$ and hence find the maximum or minimum value and the range.

(a) $y = 2x^2 - 4x$ (b) $y = -2x^2 + 8x - 3$ (c) $y = 7 + 16x - 4x^2$ (d) $y = 4x^2 + 8x - 7$

a) $y = 2x^2 - 4x = 2(x^2 - 2x) = 2[(x-1)^2 - 1] = 2(x-1)^2 - 2$

minimum value is (-2) Range is $[-2, +\infty)$

b) $y = -2x^2 + 8x - 3 = -2(x^2 - 4x) - 3 = -2[(x-2)^2 - 4] - 3$

$y = -2(x-2)^2 + 8 - 3 = -2(x-2)^2 + 5$

maximum value is 5 Range is $(-\infty, 5]$

c) $y = 7 + 16x - 4x^2 = -4(x^2 - 4x) + 7 = -4[(x-2)^2 - 4] + 7$

$y = -4(x-2)^2 + 16 + 7 = -4(x-2)^2 + 23$

maximum value is 23 Range is $(-\infty, 23)$

d) $y = 4x^2 + 8x - 7 = 4(x^2 + 2x) - 7 = 4[(x+1)^2 - 1] - 7$

$y = 4(x+1)^2 - 4 - 7 = 4(x+1)^2 - 11$

minimum value is -11 Range is $[-11, +\infty)$

QUADRATIC FUNCTIONS

4 A stone is thrown straight up from the ground. The height above the ground, $h(t)$ metres, is a function of time, t seconds ($t \geq 0$), according to the rule $h(t) = 20t - 5t^2$. Find:

(a) the domain of h **(b)** the greatest height reached.

a) The domain of h is all the values that t can take. Normally, the domain of a quadratic function is $\mathbb{R}$, however in that case, t represents the time, which cannot take negative values.
So the domain of h is $[0, +\infty)$.

b) $h(t) = 20t - 5t^2$ is the equation of a parabola, concave down as the coefficient of t^2 is negative (-5). So this parabola has a maximum, for $t = -\frac{b}{2a} = \frac{-20}{2\times(-5)} = \frac{-20}{-10} = 2\,s$.

At $t = 2s$, $h(2) = 20\times 2 - 5\times 2^2 = 40 - 20 = 20\,m$ which is the greatest height reached.

6 The sum of two numbers is 20. Find the numbers and their product if their product is a maximum.

x y

$x + y = 20$. The product of x and y is xy.

As $y = 20 - x$, the product xy can be rewritten $x(20-x)$

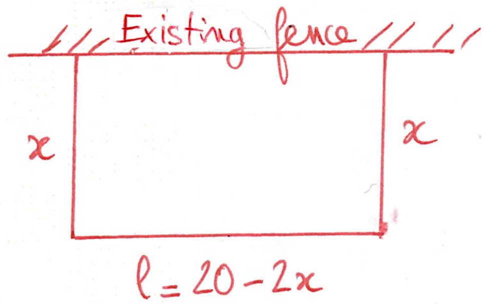
So Product $= x(20-x) = -x^2 + 20x$.

The graph of this function is a parabola, concave down as the coefficient of x^2 is negative, therefore has a maximum value, for $x_M = -\frac{b}{2a} = \frac{-20}{2\times(-1)} = \frac{-20}{-2} = 10$

when $x_M = 10$, then $y = 10$

Hence the product is maximum when $x = 10$ and $y = 10$

8 A farmer wants to make a rectangular enclosure using her existing fence as one side. If she has 20 metres of fencing material available to make the other three sides, find the area and the dimensions of the largest enclosure that can be formed.



$x + x + l = 20$

so $2x + l = 20$

so $l = 20 - 2x$

Area $= x \times l = x(20 - 2x) = -2x^2 + 20x$.

This is a parabola, concave down (as the coefficient of x^2 is negative) therefore has a maximum, for $x_n = \frac{-b}{2a} = \frac{-20}{2 \times (-2)} = \frac{-20}{-4} = 5$

when $x = 5$, $l = 20 - 2 \times 5 = 10$ m. These are the dimensions of the largest possible enclosure.

11 A machine comes in two parts, which weigh x kg and b kg respectively. The cost c of the machine (in dollars) is given by $c = 2x + b$. The earning capacity y of the machine is given by $y = x(x + b)$. If c has the fixed value 10, express y as a function of x and hence find the value of x for which y is a maximum. Find the maximum value of y.

$y = x(x + b)$ But $10 = 2x + b$ so $b = 10 - 2x$

Hence $y = x(x + (10 - 2x)) = x[x + 10 - 2x] = x[10 - x] = -x^2 + 10x$

The graph of this function is a parabola, concave down (as the coefficient of x^2 is negative), therefore has a maximum,

for $x_n = \frac{-b}{2a} = \frac{-10}{2 \times (-1)} = \frac{-10}{-2} = 5$

For this value of x, y is a maximum, and is equal

to $y = -5^2 + 10 \times 5 = -25 + 50 = 25$