DEFINITE INTEGRALS INVOLVING TRIGONOMETRIC FUNCTIONS

Summary—trigonometric integrals

$\int \sin x\,dx = -\cos x + C$	$\int \sin ax\,dx = -\frac{1}{a}\cos ax + C$	$\int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b) + C$
$\int \cos x\,dx = \sin x + C$	$\int \cos ax\,dx = \frac{1}{a}\sin ax + C$	$\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + C$
$\int \sec^2 x\,dx = \tan x + C$	$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax + C$	$\int \sec^2(ax+b)\,dx = \frac{1}{a}\tan(ax+b) + C$

Example 20

Evaluate: **(a)** $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos x\,dx$ **(b)** $\int_0^{\pi} \sin 2x\,dx$ **(c)** $\int_0^{\frac{\pi}{4}} \sec^2 x\,dx$

(d) $\int_0^{\frac{\pi}{2}} \cos\left(2x - \frac{\pi}{2}\right)dx$ **(e)** $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (\cos 2x - 3\sin x)\,dx$

Solution

(a)
$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos x\,dx = [\sin x]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$$
$$= \sin\frac{\pi}{2} - \sin\frac{\pi}{3}$$
$$= 1 - \frac{\sqrt{3}}{2}$$

(b)
$$\int_0^{\pi} \sin 2x\,dx = \left[-\frac{1}{2}\cos 2x\right]_0^{\pi}$$
$$= -\frac{1}{2}\cos 2\pi + \frac{1}{2}\cos 0$$
$$= -\frac{1}{2} + \frac{1}{2}$$
$$= 0$$

(c)
$$\int_0^{\frac{\pi}{4}} \sec^2 x\,dx = [\tan x]_0^{\frac{\pi}{4}}$$
$$= \tan\frac{\pi}{4} - \tan 0$$
$$= 1 - 0$$
$$= 1$$

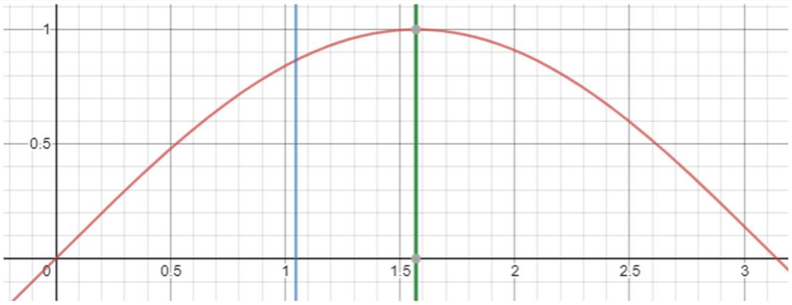
(d)
$$\int_0^{\frac{\pi}{2}} \cos\left(2x - \frac{\pi}{2}\right)dx = \left[\frac{1}{2}\sin\left(2x - \frac{\pi}{2}\right)\right]_0^{\frac{\pi}{2}}$$
$$= \frac{1}{2}\sin\frac{\pi}{2} - \frac{1}{2}\sin\left(-\frac{\pi}{2}\right)$$
$$= \frac{1}{2} + \frac{1}{2}$$
$$= 1$$

(e)
$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (\cos 2x - 3\sin x)\,dx = \left[\frac{1}{2}\sin 2x + 3\cos x\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$
$$= \left(\frac{1}{2}\sin\frac{2\pi}{3} + 3\cos\frac{\pi}{3}\right) - \left(\frac{1}{2}\sin\frac{\pi}{3} + 3\cos\frac{\pi}{6}\right)$$
$$= \frac{\sqrt{3}}{4} + \frac{3}{2} - \left(\frac{\sqrt{3}}{4} + \frac{3\sqrt{3}}{2}\right)$$
$$= \frac{3 - 3\sqrt{3}}{2}$$

Example 21

Calculate the area bounded by the curve $y = \sin x$, the x-axis and the ordinates $x = \frac{\pi}{3}$ and $x = \frac{\pi}{2}$.

Solution



$$\text{Area} = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin x \, dx$$

$$= [-\cos x]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$$

$$= \left(-\cos\frac{\pi}{2}\right) - \left(-\cos\frac{\pi}{3}\right)$$

$$= 0 + \frac{1}{2}$$

$$= \frac{1}{2}$$

. $\text{Area} = \frac{1}{2}\ \text{unit}^2$