DIVISION OF POLYNOMIALS AND THE REMAINDER THEOREM

1 Perform the following long divisions.

(a) $(3x^2 - 2x + 5) \div (x - 2)$

$$\begin{array}{r@{\,}l} & 3x + 4 \\ x-2 \,\big) & 3x^2 - 2x + 5 \\ & 3x^2 - 6x \\ \hline & 0 + 4x + 5 \\ & 4x - 8 \\ \hline & 0 + 13 \end{array}$$

$\therefore\ 3x^2 - 2x + 5 = (x-2)(3x+4) + 13$

OR $\dfrac{3x^2-2x+5}{x-2} = 3x + 4 + \dfrac{13}{x-2}$

(b) $(x^3 - x^2 + x - 1) \div (x - 1)$

$$\begin{array}{r@{\,}l} & x^2 + 1 \\ x-1 \,\big) & x^3 - x^2 + x - 1 \\ & x^3 - x^2 \\ \hline & 0 + 0 + x - 1 \\ & x - 1 \\ \hline & 0 + 0 \end{array}$$

So $x^3 - x^2 + x - 1 = (x-1)(x^2+1)$

OR $\dfrac{x^3 - x^2 + x - 1}{x-1} = x^2 + 1$

(e) $(x^3 - 27) \div (x - 3)$

$$\begin{array}{r@{\,}l} & x^2 + 3x + 9 \\ x-3 \,\big) & x^3 - 27 \\ & x^3 - 3x^2 \\ \hline & 0 + 3x^2 - 27 \\ & 3x^2 - 9x \\ \hline & 0 + 9x - 27 \\ & 9x - 27 \\ \hline & 0 + 0 \end{array}$$

So $x^3 - 27 = (x-3)(x^2+3x+9)$

OR $\dfrac{x^3-27}{x-3} = x^2 + 3x + 9$

(k) $(x^3 - 4x^2 + 2x + 3) \div (x^2 - x + 1)$

$$\begin{array}{r@{\,}l} & x - 3 \\ x^2-x+1 \,\big) & x^3 - 4x^2 + 2x + 3 \\ & x^3 - x^2 + x \\ \hline & 0 - 3x^2 + x + 3 \\ & -3x^2 + 3x - 3 \\ \hline & 0 - 2x + 6 \end{array}$$

$\therefore\ x^3 - 4x^2 + 2x + 3 = (x^2 - x + 1)(x-3) - 2x + 6$

OR

$\dfrac{x^3 - 4x^2 + 2x + 3}{x^2 - x + 1} = x - 3 + \dfrac{-2x+6}{x^2-x+1}$

5 $P(x) = x^4 - x^3 + px^2 - 4x + q$. Find p and q if $P(0) = 3$ and $P(-1) = 11$.

$P(0) = 0^4 - 0^3 + p \times 0^2 - 4 \times 0 + q = q$ so $q = 3$

$P(-1) = (-1)^4 - (-1)^3 + p(-1)^2 - 4 \times (-1) + 3$ (as $q = 3$)

so $P(-1) = 1 + 1 + p + 4 + 3 = 9 + p$

But $P(-1) = 11$, Hence $9 + p = 11$ $\therefore p = 2$

6 $P(x) = ax^3 - 2x^2 + bx + c$. Find a, b and c if $P(0) = 12$, $P(-1) = 3$ and $P(2) = 36$.

$P(0) = a \times 0^3 - 2 \times 0^2 + b \times 0 + c = c$ $\therefore c = 12$

$P(-1) = a \times (-1)^3 - 2(-1)^2 - b + 12 = -a - 2 - b + 12$

$\therefore P(-1) = -a - b + 10$ So $-a - b + 10 = 3$

Hence $a + b = 7$

$P(2) = a\, 2^3 - 2 \times 2^2 + b \times 2 + 12 = 8a - 8 + 2b + 12$

$P(2) = 8a + 2b + 4$ So $8a + 2b + 4 = 36$

Hence $4a + b = 16$

So $\begin{cases} a + b = 7 \\ 4a + b = 16 \end{cases}$ By elimination, $3a = 16 - 7 = 9$

$\therefore a = 3$ and therefore $b = 4$

$a = 3,\ b = 4,\ c = 12$

7 Using the remainder theorem, find the remainder when:

(a) $x^3 + 3x^2 + 2x - 7$ is divided by $(x + 2)$ (b) $3x^2 + 7x - 2$ is divided by $(x - 4)$

Remainder Theorem: if a polynomial $P(x)$ is divided by $(x-a)$ until the remainder R does not contain x, then $R = P(a)$.

a) $P(-2) = (-2)^3 + 3 \times (-2)^2 + 2 \times (-2) - 7 = -8 + 12 - 4 - 7 = -7$

So when (x^3+3x^2+2x-7) is divided by $(x+2)$, the remainder is (-7)

b) $P(4) = 3 \times 4^2 + 7 \times 4 - 2 = 48 + 28 - 2 = 74$

So when $(3x^2+7x-2)$ is divided by $(x-4)$, the remainder is 74

10 When the polynomial $x^3 + 3x^2 - mx + n$ is divided by $(x + 2)$ the remainder is 9; when divided by $(x - 3)$ the remainder is 49. Find m and n.

$P(-2) = (-2)^3 + 3 \times (-2)^2 - m \times (-2) + n$

$ = -8 + 12 + 2m + n$

$ = 2m + n + 4$ $\quad \therefore 2m + n + 4 = 9$

$\therefore 2m + n = 5$

$P(3) = 3^3 + 3 \times 3^2 - m \times 3 + n$

$ = 27 + 27 - 3m + n$

$ = 54 - 3m + n$ $\quad \therefore 54 - 3m + n = 49$

$\therefore -3m + n = -5$

We now need to solve simultaneously:

$\begin{cases} 2m + n = 5 \\ -3m + n = -5 \end{cases}$ By elimination: $5m = 10$ so $m = 2$

and then $n = 5 - 2 \times 2 = 1$

So $m = 2$ and $n = 1$

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13 When $2x^3 + 7x^2 + ax + b$ is divided by $(x - 3)$ the remainder is 120; when divided by $(x + 1)$ the remainder is -8. Find the values of a and b.

Using the remainder theorem:

$P(3) = 2 \times 3^3 + 7 \times 3^2 + a \times 3 + b$

$P(3) = 54 + 63 + 3a + b$

$P(3) = 117 + 3a + b$

Therefore $117 + 3a + b = 120$

$\therefore \quad 3a + b = 3$

$P(-1) = 2 \times (-1)^3 + 7 \times 1^2 + a \times (-1) + b$

$\quad = -2 + 7 - a + b$

$\quad = 5 - a + b$

Therefore $5 - a + b = -8$

$\therefore \quad -a + b = -13$

We now need to solve simultaneously:

$$\begin{cases} 3a + b = 3 \\ -a + b = -13 \end{cases}$$

By elimination: $4a = 16$ $\quad \therefore a = 4$

Then $b = 3 - 3 \times 4 = -9$

Therefore, in conclusion, $a = 4$ and $b = -9$